

DESIGN OF SLABS : Two-Way Slab

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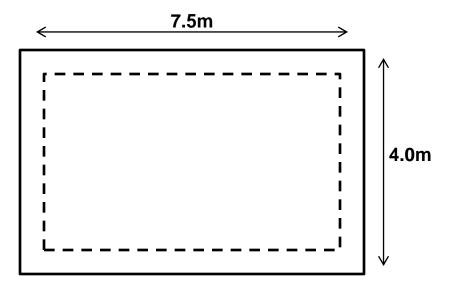
Design 1



THA

Simply Supported (Two-Way Spanning Slab)

Design a reinforced concrete slab for a room measuring 7.5m x 5.0m. The slab carries a variable action of 2.5kN/m² and permanent action due to finishes of 1.0kN/m². The characteristic material strength are $f_{ck}=25$ N/mm² and $f_{yk}=500$ N/mm². The slab can be considered simply supported on all four edges with corners free to lift. The slab is inside buildings which subjected to 1.5 hours fire resistance and 50 years design life.





A slab simply supported on its four sides will deflect about both axes under load and the corner will tend to lift and curl up from the support, causing torsional moments. When no provision has been made to prevent this lifting or to resist the torsion, then the moment coefficient of *Table 3.13 BS8110: 1997* may be used and the maximum moments as given by:

 $m_{sx} = lpha_{sx} n l_x^2$ $m_{sy} = lpha_{sy} n l_x^2$

where;

n = total ultimate load per unit area l_x = length of shorter side l_y = length of longer side α_{sx} and α_{sy} are the moment coefficient

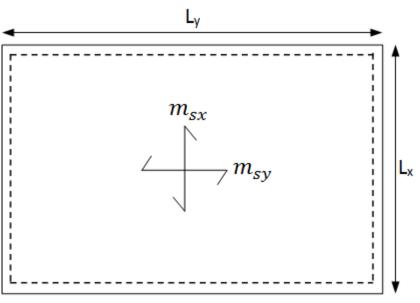


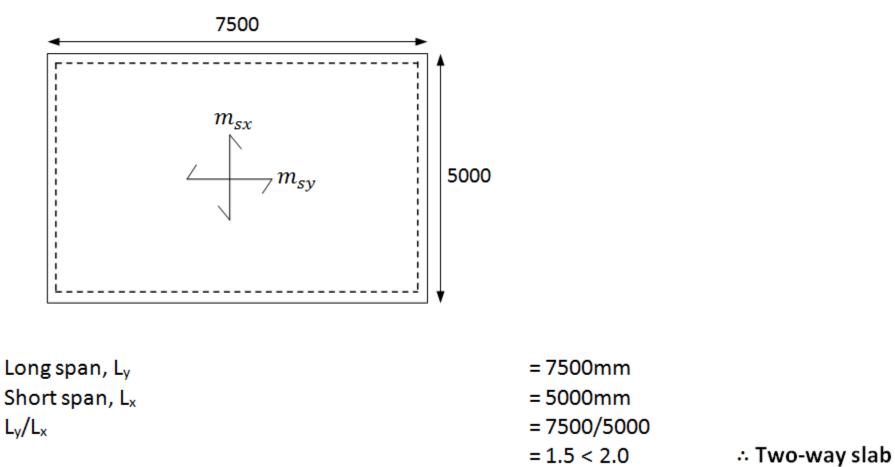


Table 3.13 BS8110: 1997

L _y /L _x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_{sx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
α_{sy}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029



1.0 SPECIFICATION





Characteristic actions: Permanent, g_k Variable, q_k Design life Fire resistance Exposure classes Materials: Characteristic strength of concrete, f_{ck} Characteristic strength of steel, f_{yk} Unit weight of reinforced concrete Assumed: Ø_{bar}

2.0 SLAB THICKNESS

Min. thickness for fire resistance= 120mmEstimated thickness for deflection control, h= Lx/26= 192mmUse, h = 200mm

- = 1.2kN/m² (excluding selfweight)
- = 2.5kN/m²
- = 50 Years
- = R90
- = XC1
- = 25N/mm²
- = 500N/mm²
- = 25kN/m³
- = 10mm



3.0 DURABILITY, FIRE AND BOND REQUIREMENTS

Min. concrete cover regard to bond, C _{min,} Min. concrete cover regard to durability, Min. required axis distance for R90, a	-
Min. concrete cover regard to fire	
$C_{min,fire} = a - \phi_{bar}/2$	= 15-(10/2)
	= 10mm
Allowance in design for deviation, $\Delta Cdev$	= 10mm
Nominal cover,	
$C_{nom} = C_{min} + \Delta C_{dev}$	= 15+10
	= 25mm
∴ C _n	_{om} = 25mm



Table 5.8: Minimum dimensions and axis distances for reinforced and prestressed concrete simply supported one-way and two-way solid slabs

Standard fire resistance	Minimum dimensions (mm)				
	slab	axis-distance a			
	thickness h _s (mm)	one way	two way: $l_y/l_x \le 1.5$ $1.5 < l_y/l_x \le$		
1	2	3	4	5	
REI 30	60	<mark>10*</mark>	10*	10*	
REI 60	80	20	10*	15*	
REI 90	100	30	15*	20	
REI 120	120	40	20	25	
REI 180	150	55	30	40	
REI 240	175	65	40	50	

The axis distance *a* in Column 4 and 5 for two way slabs relate to slabs supported at all four edges. Otherwise, they should be treated as one-way spanning slab.

* Normally the cover required by EN 1992-1-1 will control.



4.0 LOADING AND ANALYSIS

Slab self-weight	= 0.2x25	
	= 5.0kN/m ²	
Permanent load (excluding self-weight)	= 1.2kN/m ²	
Characteristic permanent action, G _k	$= 6.2 k N/m^2$	
$Characteristic variable action, Q_k$	$= 2.5 kN/m^2$	
Design action, n _d	= 1.35G _k +1.5Q _k	
	= 1.35(6.2) + 1.5(2.5)	
	= 12.12kN/m ²	
	= 11.76kN/m	
Moment:		
Short span, $M_{sx} = \alpha_{sx} n L_x^2$	= 0.104x12.12x5.02	
	= 31.51kNm/m	
Long span, $M_{sy} = \alpha_{sy} n L_x^2$	= 0.046x12.12x5.02	= 13.94kNm/m
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5.0 MAIN REINFORCEMENT

Effective depth: $d_x = h-C_{nom}-0.5Ø_{bar}$

 $d_y = h-C_{nom}-1.5Ø_{bar}$

Short span:

 M_{sx} K = M/bd²f_{ck}

- = 200-30-(0.5x10)
- = 170mm
- = 200-30-(1.5x10)
- = 160mm
- = 31.5kNm/m = 31.5x10⁶/(1000x170²x25)
- $= 0.044 < K_{bal} = 0.167$

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$

= 0.96d \geq 0.95d

Use 0.95d



 $As = M/0.87 f_{yk}z$

= 31.5x10⁶/(0.87x500x0.95x170) = 449mm²/m

H10-175 (top) (449mm²/m)

Long span:

 M_{sx} K = M/bd²f_{ck} = 13.9kNm/m = 13.9x10⁶/(1000x160²x25) = 0.022 < K_{bal}=0.167

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$

= 0.98d \geq 0.95d

Use 0.95d



 $As = M/0.87 f_{yk}z$

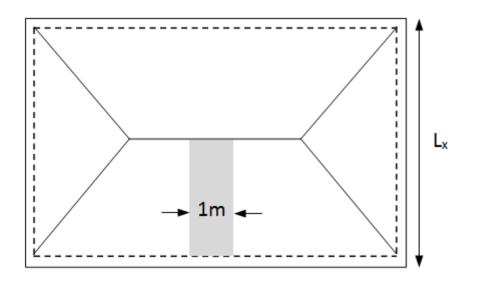
= 13.9x10⁶/(0.87x500x0.95x170) = 211mm²/m

H10-325 (bot) (242mm²/m)

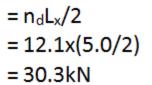
Min. and max. reinforcement area, A _{s,min} = 0.26(f _{ctm} /f _{yk})bd	Use 0.0013bd	= 0.26x(2.56/500)bd = 0.0013bd > 0.0013bd = 0.0013x1000x170 = 227mm ² /m	
$A_{s,max} = 0.04A_c = 0.04bh$		= 0.04x1000x200 = 8000mm ² /m	



6.0 SHEAR



Max. design shear force, V_{Ed}





Design shear resistance, $k = 1+(200/d)^{1/2} \le 2.0$

 $\rho^1 = A_{s1}/bd \le 0.02$

 $V_{Rd,c} = [0.12k(100\rho^1 f_{ck})^{1/3}]bd$

 $V_{min} = [0.35k^{3/2}f_{ck}^{1/2}]bd$

 $= 1+(200/170)^{1/2} \\ = 2.08 \ge 2.0$

 $= 449/(1000 \times 170) \\ = 0.0026 \le 0.02$

= [0.12x2.0(100x0.0026x30)^{1/3}]x1000x170 = 76535N = 76.5kN

= [0.0035x2.0^{3/2}x25^{1/2}]x1000x170 = 84146N = 84.1kN

V_{Rd,c}; V_{min} > V_{Ed} (OK)



7.0 DEFLECTION

Percentage of required tension reinforcement, $ ho$ =A _{s,req} /bd	= 449/(1000x170) = 0.0026
Reference reinforcement ratio, $\rho_0 = (f_{ck})^{1/2} \times 10^{-3}$	= (25) ^{1/2} x10 ⁻³ = 0.0050
Percentage of required compression reinforcement $\rho' {=} {\rm A}_{\rm s', req} / {\rm bd}$	= 0/(1000x139) = 0.000
Factor for structural system, K = 1.0	



$$\frac{l}{d} = K \left[11 + 1.5\sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}} \left(\frac{\rho_0}{\rho} - 1\right) \right]^{3/2}$$

Modification factor for span less than 7m

Modification factor for steel area provided

= 1.0(11+14.2+13.55) = 38.8

 $= A_{s,prov}/A_{s,req}$ = 449/449 = 1.0 \leq 1.5

= 1.0

Therefore allowable span-effective depth ratio

= (I/d)_{allowable} = 38.8x1.0x1.0 = 38.8

Actual span-effective depth

= (I/d)_{actual} = 5000/170 = 29.4 < (I/d)_{allowable}

(OK)



8.0 CRACKING

h = 200mm < 200mm

Main bar, S_{max,slabs} = 3h or 400mm

= 600mm > 400mm

Max. bar spacing = 325mm $\leq S_{max,slab}$ = 400mm

Secondary bar, S_{max,slabs} = 3.5h or 450mm

= 700mm > 450mm

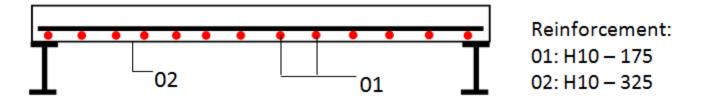
Max. bar spacing = 350mm \leq S_{max,slab} = 450mm

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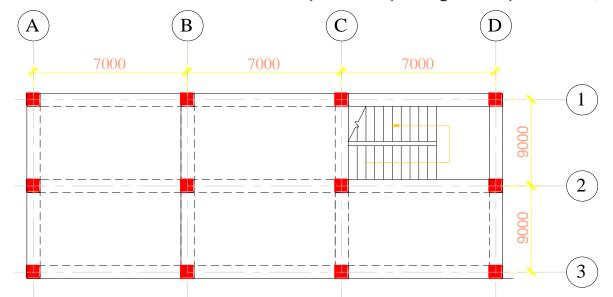


Design 2

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Restrained (Two-Way Spanning Slab)

The figure below shows part of the first floor plan of a reinforced concrete office building. During construction, slabs and beams are cast together. The finishes, ceiling and services form a characteristic permanent action of 1.5kN/m² (excluding self-weight). The characteristic variable action is 4.0kN/m². The floor is inside enclosed building and subjected to 1 hour fire resistance requirements. The construction materials consist of grade C25 concrete and 500 steel. Based on the information provided, design slab panel B-C/1-2.



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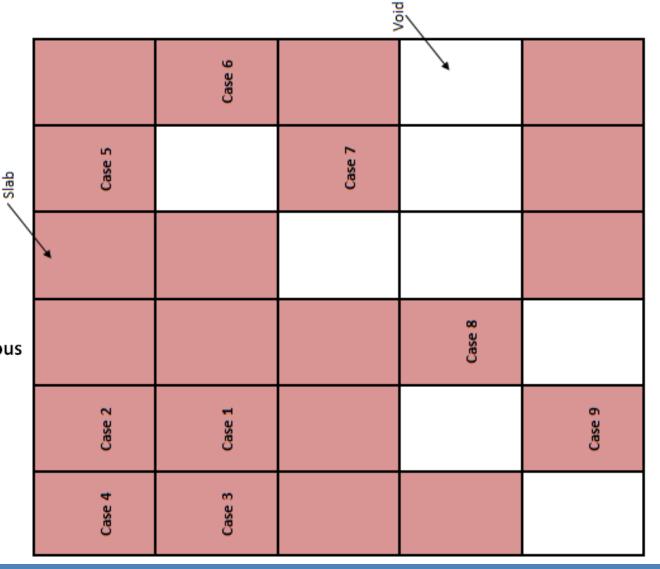
When the slab are provided with different edge conditions like fixed or continuous edges, the maximum moments per unit width are given by:

 $m_{sx} = \beta_{sx} n l_x^2$ $m_{sy} = \beta_{sy} n l_x^2$ where; m_{sx} n = total ultimate load per unit area l_x = length of shorter side 11 m_{sy} $_7m_{sy}$ l_{v} = length of longer side 11 eta_{sx} and eta_{sy} are the moment coefficient from Table 3.14 BS8110: Part 1: 1997 $\backslash m_{sx}$ 11 11



There are nine different types of support conditions to be considered which relate to the particular support/restraint provided on edge of individual slabs.

Case 1: Four edges continuous Case 2: One short edge discontinuous Case 2: One long edge discontinuous Case 4: Two adjacent edge discontinuous Case 5: Two short edge discontinuous Case 6: Two long edges discontinuous Case 7: Three edges discontinuous (one long edge continuous) Case 8: Three edges discontinuous (one short edge continuous) Case 9: Four edges discontinuous





Type of panel and moments Short span coefficients, β_{zz} Long span considered coefficients, Values of l_{y}/l_{z} β_{zy} for all values of l_{μ}/l_{μ} 1.0 1.1 1.2 1.4 1.0 1.75 2.0 1.8 Interior panels Negative moment at 0.031 0.037 0.0420.046 0.050 0.053 0.059 0.063 0.032 continuous edge Positive moment at mid-span 0.028 0.037 0.024 0.0240.032 0.035 0.0400.0440.048 One short edge discontinuous Negative moment at 0.039 0.044 0.048 0.052 0.055 0.058 0.063 0.067 0.037 continuous edge Positive moment at mid-span 0.029 0.033 0.036 0.039 0.041 0.043 0.0470.050 0.028 One long edge discontinuous Negative moment at 0.039 0.049 0.056 0.062 0.068 0.073 0.082 0.089 0.037 continuous edge Positive moment at mid-span 0.036 0.0420.051 0.055 0.062 0.067 0.028 0.030 0.047Two adjacent edges discontinuous Negative moment at 0.0470.056 0.063 0.069 0.0740.078 0.087 0.093 0.045 continuous edge 0.036 Positive moment at mid-span 0.0420.0470.051 0.055 0.059 0.065 0.070 0.034 Two short edges discontinuous Negative moment at 0.046 0.050 0.0540.057 0.060 0.0620.067 0.070 continuous edge Positive moment at mid-span 0.034 0.038 0.040 0.043 0.045 0.0470.050 0.053 0.034 Two long edges discontinuous Negative moment at 0.045continuous edge Positive moment at mid-span 0.034 0.046 0.056 0.065 0.0720.078 0.091 0.100 0.034 Three edges discontinuous (one long edge continuous) Negative moment at 0.057 0.065 0.071 0.076 0.081 0.084 0.092 0.098 continuous edge Positive moment at mid-span 0.043 0.048 0.053 0.057 0.060 0.063 0.069 0.0740.044Three edges discontinuous (one short edge continuous) Negative moment at 0.058 continuous edge Positive moment at mid-span 0.042 0.0540.063 0.071 0.078 0.084 0.096 0.105 0.044Four edges discontinuous Positive moment at mid-span 0.055 0.065 0.074 0.081 0.087 0.092 0.103 0.056 0.111

Table 3.14 — Bending moment coefficients for rectangular panels supported on four sides with

provision for torsion at corners



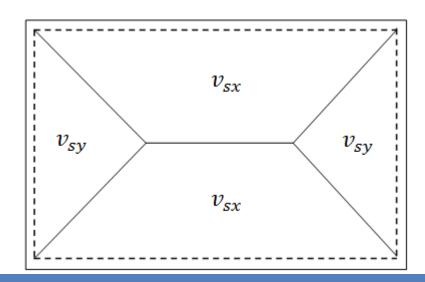
Shear force and actions on supporting beams

The design shear forces of slab or loads on beams which supported the slab can be evaluated using the equations below:

 $v_{sx} = \beta_{vx} n l_x$ $v_{sy} = \beta_{vy} n l_x$

where;

n = total ultimate load per unit area l_x = length of shorter side l_y = length of longer side β_{vx} and β_{vy} are the shear coefficient from Table 3.115 BS8110: Part 1: 1997





Type of panel and location	β_{m} for values of $l J l_{m}$				β _{wy}				
	1.0	1.1	1.2	1.8	1.4	1.0	1.70	2.0	1 W
Four edges continuous	1.0			1.0			1.10		
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	—	_	_	—	_	_	—	—	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	-
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
Two short edges discontinuous									
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	_
Discontinuous edge	—	—	-	—	—	—	—	—	0.26
Two long edges discontinuous									
Continuous edge	<u> </u>	-	-	—	-	-	—	 _	0.40
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	-
Three edges discontinuous (one long edge discontinuous)									
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	—
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Three edges discontinuous (one short edge discontinuous)									
Continuous edge	—	-	-	-	-	—	-	-	0.45
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Four edges discontinuous	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
Discontinuous edge	0.55	0.50	0.59	0.41	0.45	0.45	0.48	0.50	0.55

Table 3.15 — Shear force coefficient for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners



1.0 SPECIFICATION

Long span, L _y Short span, L _x L _y /L _x	= 7000mm = 4000mm = 7000/4000 = 1.75 < 2.0		
∴ Two-way slab			
Characteristic actions:			
Permanent, g _k Variable, q _k C Assumed: Ø _{bar}	= 1.5kN/m ² (excluding = 4.0kN/m ² = 10mm	selfweight)	
Fire resistance	= R60		
Exposure classes	= XC1		
Materials:			
Characteristic strength of concrete, <i>f_{ck}</i> Characteristic strength of steel, <i>f_{yk}</i> Unit weight of reinforced concrete	= 25N/mm ² = 500N/mm ² = 25kN/m ³	Assumed: Ø _{bar} = 10mm	



2.0 SLAB THICKNESS

Min. thickness for fire resistance Estimated thickness for deflection control, h

= 80mm = L_x/35 (Based on Table 5.4N, L/30) = 4000/35 = 114mm

Use, h = 125mm



3.0 DURABILITY, FIRE AND BOND REQUIREMENTS

Min. concrete cover regard to bond Min. concrete cover regard to dura Min. required axis distance for R60,	bility, C _{min, dur}	= 10mm = 15 = 15mm
Min. concrete cover regard to fire $C_{min, fire} = a - \emptyset_{bar}/2$		= 15-(10/2) = 10mm
Allowance in design for deviation, A	\Cdev	= 10mm
Nominal cover, $C_{nom} = C_{min} + \Delta C_{dev}$		= 15+10 = 25mm
	∴ C _{nom} = 25mm	



4.0 LOADING AND ANALYSIS

Slab self-weight	$= 0.125 \times 25$ = 3.13kN/m ²
Permanent load (excluding self-weight)	$= 1.50 \text{kN/m}^2$
Characteristic permanent action, G _k	= 4.63kN/m ²
Characteristic variable action, Q _k	$= 4.00 \text{kN/m}^2$
Design action, n _d	= $1.35G_k+1.5Q_k$ = $1.35(4.63) + 1.5(4.00)$ = $12.24kN/m^2$
Moment:	
Short span, $M_{sx1} = \beta_{sx1} n_d L_x^2$	= 0.065x12.24x4.0 ²
	= 12.7kNm/m
$M_{sx2} = \beta_{sx2} n_d L_x^2$	$= 0.087 \times 12.24 \times 4.0^{2}$
	= 17.0kNm/m



Long span, $M_{sy1} = \beta_{sy1} n_d L_x^2$

$$M_{sy2} = \beta_{sy2} n_d L_x^2$$

<u>Shear</u>: Short span, $V_{sx1} = \beta_{vx1} n_d L_x$

$$V_{sx2} = \beta_{vx2} n_d L_x$$

Long span, $V_{sy1} = \beta_{vy1} n_d L_x$

$$V_{sy2} = \beta_{vy2} n_d L_x$$

- = 0.034x12.24x4.0² = 6.66kNm/m = 0.045x12.24x4.0² = 8.82kNm/m
- = 0.57x12.24x4.0 = 27.9kNm/m = 0.38x12.24x4.0 = 18.6kNm/m
- = 0.40x12.24x4.0
- = 19.6kNm/m
- = 0.26x12.24x4.0
- = 12.7kNm/m



5.0 MAIN REINFORCEMENT

Effective depth: $d_x = h-C_{nom}-0.5 Ø_{bar}$

 $d_y = h-C_{nom}-1.5 Ø_{bar}$

Short span (Midspan): M_{sx} K = M/bd²f_{ck} = 125-20-(0.5x10) = 95mm = 125-20-(1.5x10) = 85mm

= 12.7kNm/m = 12.7x10⁶/(1000x95²x25) = 0.056 < K_{bal}=0.167

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$





 $As = M/0.87 f_{yk}z$

= 12.7x10⁶/(0.87x500x0.95x95) = 325mm²/m

H10-200 (bot) (393mm²/m)

<u>Short span (Support)</u> :	
M _{sx}	= 17.0kNm/m
$K = M/bd^2 f_{ck}$	$= 17.0 \times 10^{6} / (1000 \times 95^{2} \times 25)$
	= 0.076 < K _{bal} =0.167

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$

= 0.93d \leq 0.95d

Use 0.93d



 $As = M/0.87 f_{yk}z$

= 17.0x10⁶/(0.87x500x0.93x95) = 444mm²/m

H10-175 (top) (449mm²/m)

Long span (Midspan):
M _{sy}
$K = M/bd^2 f_{ck}$

= 6.7kNm/m = 6.7x10⁶/(1000x85²x25) = $0.037 < K_{bal} = 0.167$

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$

 $= 0.97d \ge 0.95d$

Use 0.95d



 $As = M/0.87 f_{yk}z$

= 6.7x10⁶/(0.87x500x0.95x85) = 190mm²/m

H10-350 (bot) (224mm²/m)

Long span (Support):
M _{sy}
$K = M/bd^2 f_{ck}$

= 8.8kNm/m = 8.8x10⁶/(1000x85²x25) = $0.039 < K_{bal} = 0.167$

: Compression reinforcement is not required

 $z = d[0.5 + \sqrt{0.25 - K/1.134}]$

= 0.96d \geq 0.95d

Use 0.95d



 $= 8.8 \times 10^{6} / (0.87 \times 500 \times 0.95 \times 85)$ $As = M/0.87 f_{yk}z$ $= 225 mm^{2}/m$ H10-325 (top) $(242 mm^{2}/m)$ Min. and max. reinforcement area, = 0.26x(2.56/500)bd $A_{s,min} = 0.26(f_{ctm}/f_{yk})bd$ = 0.0013bd > 0.0013bd Use 0.0013bd $= 0.0013 \times 1000 \times 95$ $= 127 mm^{2}/m$ $= 0.04 \times 1000 \times 125$ $A_{s,max} = 0.04A_c = 0.04bh$ $= 5000 \text{mm}^2/\text{m}$ H10-425 $(185 mm^{2}/m)$



6.0 SHEAR

Max. design shear force, V _{Ed}	= 27.9kN
Design shear resistance,	
$k = 1 + (200/d)^{1/2} \le 2.0$	$= 1 + (200/95)^{1/2}$
	= 2.45 ≥ 2.0
$ ho^1$ = A _{s1} /bd \leq 0.02	= 449/(1000x95)
	$= 0.0047 \le 0.02$
$V_{Rd,c} = [0.12k(100\rho^1 f_{ck})^{1/3}]bd$	$= [0.12x2.0(100x0.0047x25)^{1/3}]x1000x95$
	= 51925N = 51.9kN
$V_{min} = [0.35k^{3/2}f_{ck}^{1/2}]bd$	= [0.0035x2.0 ^{3/2} x25 ^{1/2}]x1000x95
	= 47023N = 47.0kN VRd,c; Vmin > VEd (OK)



7.0 DEFLECTION

Percentage of required tension reinforcement, $ ho = A_{s,req}/bd$	= 325/(1000x95) = 0.0034
Reference reinforcement ratio, $\rho_0 = (f_{ck})^{1/2} \times 10^{-3}$	= (25) ^{1/2} x10 ⁻³ = 0.0050
Percentage of required compression reinforcement $\rho' = {\rm A_{s',req}/bd}$	= 0/(1000x139) = 0.000
Factor for structural system, K = 1.3	

Factor for structural system, K = 1.3 $\rho < \rho_0$ \therefore use equation (2)



$$\frac{l}{d} = K \left[11 + 1.5 \sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3.2 \sqrt{f_{ck}} \left(\frac{\rho_0}{\rho} - 1 \right) \right]^{3/2}$$

Modification factor for span less than 7m

Modification factor for steel area provided

Therefore allowable span-effective depth ratio

Actual span-effective depth

- = 1.3(11+11+5) = 35 = 1.0 = A_{s,prov}/A_{s,req} = 393/325 = 1.21 ≤ 1.5
- = (I/d)_{allowable} = 35x1.0x1.21 = 42.3
- = (I/d)_{actual} = 4000/95 = 42.1 < (I/d)_{allowable} (OK)



8.0 CRACKING

h = 125mm < 200mm

Main bar, S_{max,slabs} = 3h or 400mm

= 375mm < 400mm

Max. bar spacing = 350mm $\leq S_{max,slab} = 375$ mm

Secondary bar, S_{max,slabs} = 3.5h or 450mm

= 438mm < 450mm

Max. bar spacing = 425mm $\leq S_{max,slab} = 438$ mm

(OK)

(OK)