BEAM DESIGN

Introduction

- The purposes of this chapter is to compile the design principles that have been previously discussed in order to form a complete design procedures of reinforced concrete beam.

- Basically, beam is the structural element which subjected to transverse load in the forms bending moment, shear force and torsion. Therefore beam is designed to resist all that particular factors.

- Beside of that, beams are also designed to fulfill the serviceability requirements in order to produce an adequate and safe design.

Introduction

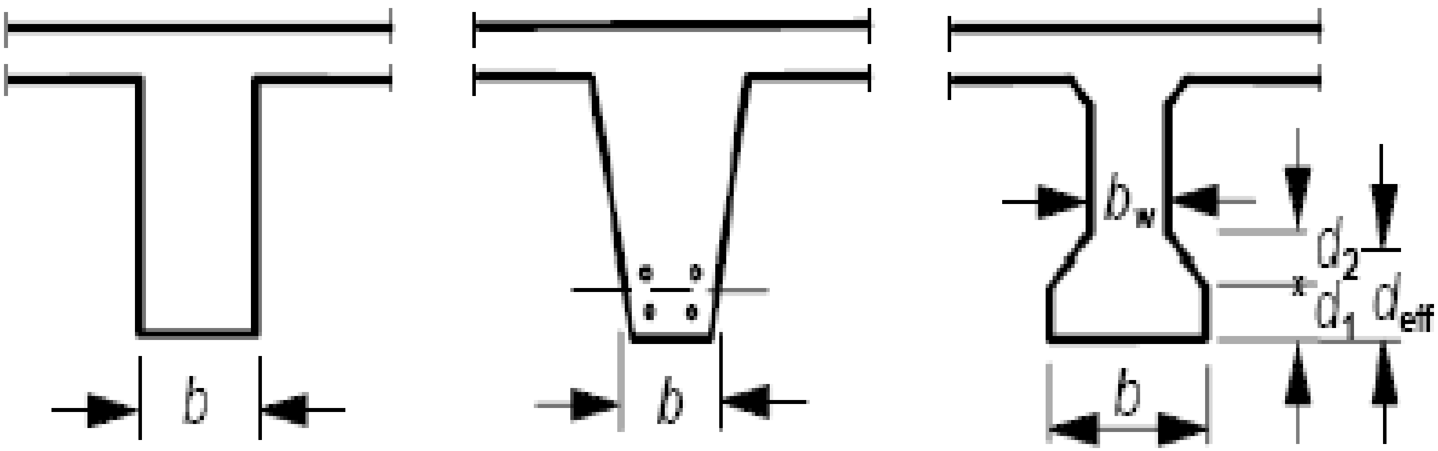
- The complete beam design procedures can be state as follows;
  - ❑ Determine design life
  - ❑ Determine preliminary beam sizing
  - ❑ Estimate actions on beam
  - ❑ Asses durability requirement and determine concrete strength
  - ❑ Determine nominal cover for durability, fire and bond requirement
  - ❑ Analysis structure to obtain critical moments and shear forces
  - ❑ Design of flexural reinforcement
  - ❑ Design of shear reinforcement
  - ❑ Verify deflection
  - ❑ Verify cracking
  - ❑ Detailing

Beam Sizing

- From structural point of view, the selection of beam sizes is often dictated by deflection control criteria.
- Span to overall depth (L/h) ratios of 13 to 18 are generally found to be economical in the case of simply supported and continuous beam.
- The recommended ratio of width to overall depth (b/h) in rectangular beam section is in range of 0.3 – 0.6.
- Beside of that, the beam sizes also control by an architectural detailing and fire resistance requirement.

Beam Sizing

EC 2 Part 1-2, gives a method for determining the minimum dimension of beams for fire resistance requirements.



(a) Constant width (b) Variable width (c) I - section

Beam Sizing

Minimum dimensions for beam for fire resistance

Standard fire resistance		Minimum dimensions (mm)							
		Possible combinations of a and b_{min} where a is the average axis distance and b_{min} is the width of the beam							
		Simply supported beams				Continuous beams			
		A	B	C	D	E	F	G	H
R60	b_{min} = a =	120 40	160 35	200 30	300 25	120 25	200 12[a]		
R90	b_{min} = a =	150 55	200 45	300 40	400 35	150 35	250 25		
R120	b_{min} = a =	200 65	240 60	300 55	500 50	200 45	300 35	450 35	500 30
R240	b_{min} = a =	280 90	350 80	500 75	700 70	280 75	500 60	650 60	700 50

- For fire resistance R120-R240, the width of the beam at the first intermediate support should be at least that in column F, if both the following conditions exist:

  i) there is no fixity at the end support, and

  ii) the acting shear at normal temperature $V_{sd} > 0.67\ V_{Rd,max}$

Fig.3.5, Cl.7.3.3(2), Cl.7.3.1, Table7.2N

- Design of RC beam for flexure are based on the following steps:

Design procedures of rectangular beam:-

1. Calculate $K = \dfrac{M}{bd^2 f_{ck}}$

2. Calculate $K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$

where $\delta = \dfrac{\text{momen at section after redistribution}}{\text{momen at section before redistribution}} \leq 1.0$

Design for flexure

If $K \leq K_{bal}$, compression reinforcement is not required, and

i. $z = d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right]$

ii. $A_s = \dfrac{M}{0.87 f_{yk} z}$

If $K > K_{bal}$, compression reinforcement is required, and

i. $z = d\left[0.5 + \sqrt{(0.25 - K_{bal}/1.134)}\right]$

ii. $x = (d - z)/0.4$

iii. Check d'/x

Design for flexure

$$A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')} \quad \text{if } d'/x \leq 0.38 \text{ or}$$

f_{sc} → $0.87 f_{yk}$

$$A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{f_{sc} (d - d')} \quad \text{if } d'/x > 0.38$$

where $f_{sc} = 700(1 - d'/x)$

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A_s' \left(\frac{f_{sc}}{0.87 f_{yk}} \right)$$

Check A_{smin} and A_{smax} -----> Cl.9.2.1.1

Design Procedures for Flange Section:-

1. Calculate $M_f = 0.567 f_{ck} b h_f (d - 0.5 h_f)$

2. If $M \leq M_f$, neutral axis in the flange

   i. $K = \dfrac{M}{b d^2 f_{ck}}$

   ii. $z = d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right]$

   iii. $A_s = \dfrac{M}{0.87 f_{yk} z}$

Design for flexure

If $M > M_f$, neutral axis in the flange

i. Calculate $\beta_f = 0.167\frac{b_w}{b} + 0.567\frac{h_f}{d}\left(1-\frac{b_w}{b}\right)\left(1-\frac{h_f}{2d}\right)$

ii. Calculate $M_{bal} = \beta_f f_{ck} b d^2$

iii. Compare M and M_{bal}

If $M \leq M_{bal}$, compression reinforcement is not required.

i. $A_s = \frac{M + 0.1 f_{ck} b_w d (0.36d - h_f)}{0.87 f_{yk} (d - 0.5h_f)}$

Design for flexure

If $M > M_{bal}$, compression reinforcement is required.

i. $A_s' = \dfrac{(M - M_{bal})}{0.87 f_{yk}(d - d')}$

ii. $A_s = \dfrac{0.167 f_{ck} b_w d + 0.567 f_{ck} h_f (b - b_w)}{0.87 f_{yk}} + A_s'$

Design for flexure

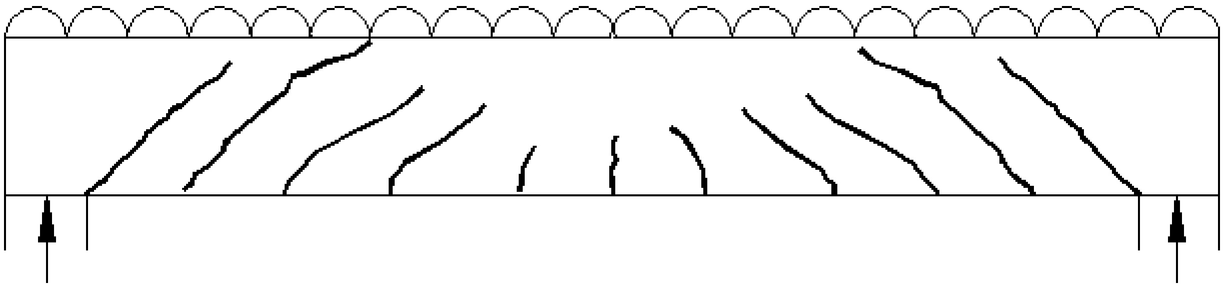
- Area of reinforcement bars

Table 1: Cross Sectional Area (mm^2) according to Size and Numbers of Bar

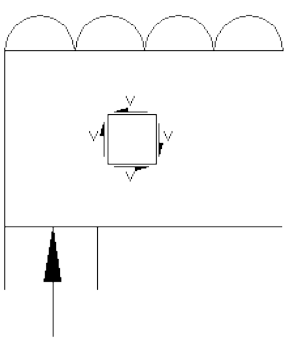
Bar Size (mm)	Number of bar								Perimeter (mm)
	1	2	3	4	5	6	7	8	
6	28.3	56.6	84.9	113	141	170	198	226	18.9
8	50.3	101	151	201	251	302	352	402	25.1
10	78.6	157	236	314	393	471	550	629	31.4
12	113	226	339	453	566	679	792	905	37.7
16	201	402	603	805	1006	1207	1408	1609	50.3
20	314	629	943	1257	1571	1886	2200	2514	62.9
25	491	982	1473	1964	2455	2946	3438	3929	78.6
32	805	1609	2414	3218	4023	4827	5632	6437	100.6
40	1257	2514	3771	5029	6286	7543	8800	10057	125.7

Design for shear

Cl.6.2.1(8), Cl.6.3(1&3), Cl.6.3.(3), Cl.6.2.3(3), Cl.9.2.2(5&6)



- **Where BM is greatest:** Cracks are caused by Bending Stress in the tension zone
- **Where SF is greatest:** Cracks are caused by Diagonal Tension / Diagonal Compression due to complementary shears



Design for shear

- EC 2 introduces the strut inclination method for shear capacity checks. In this method the shear is resisted by concrete struts acting in compression and shear reinforcement acting in tension.

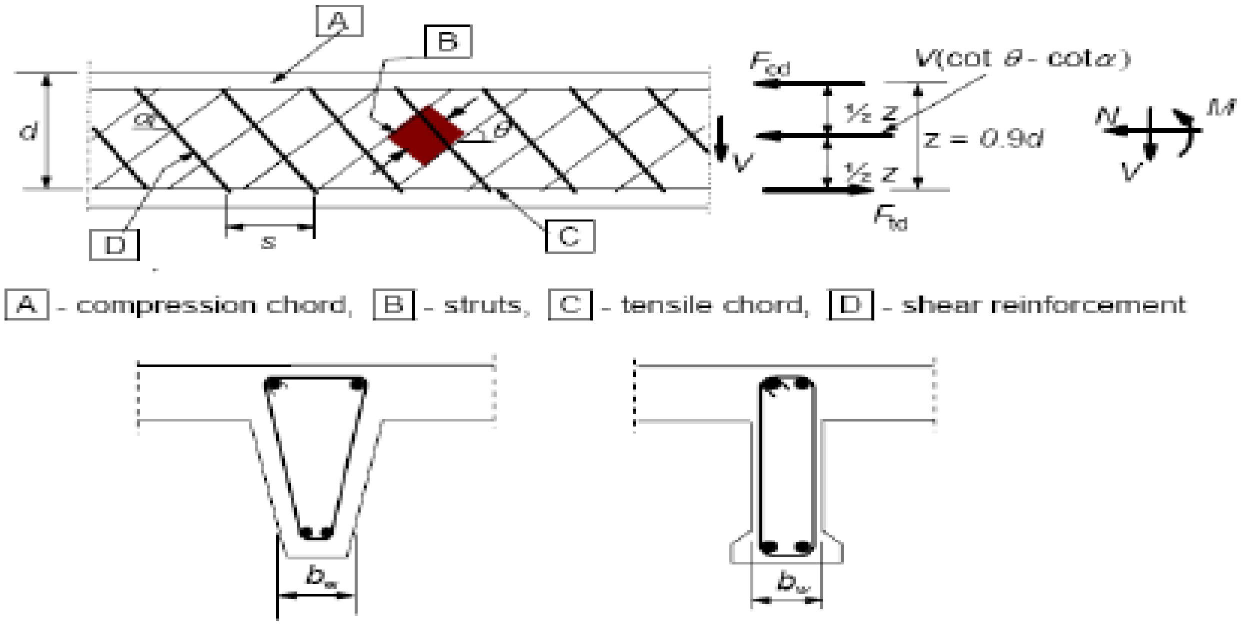
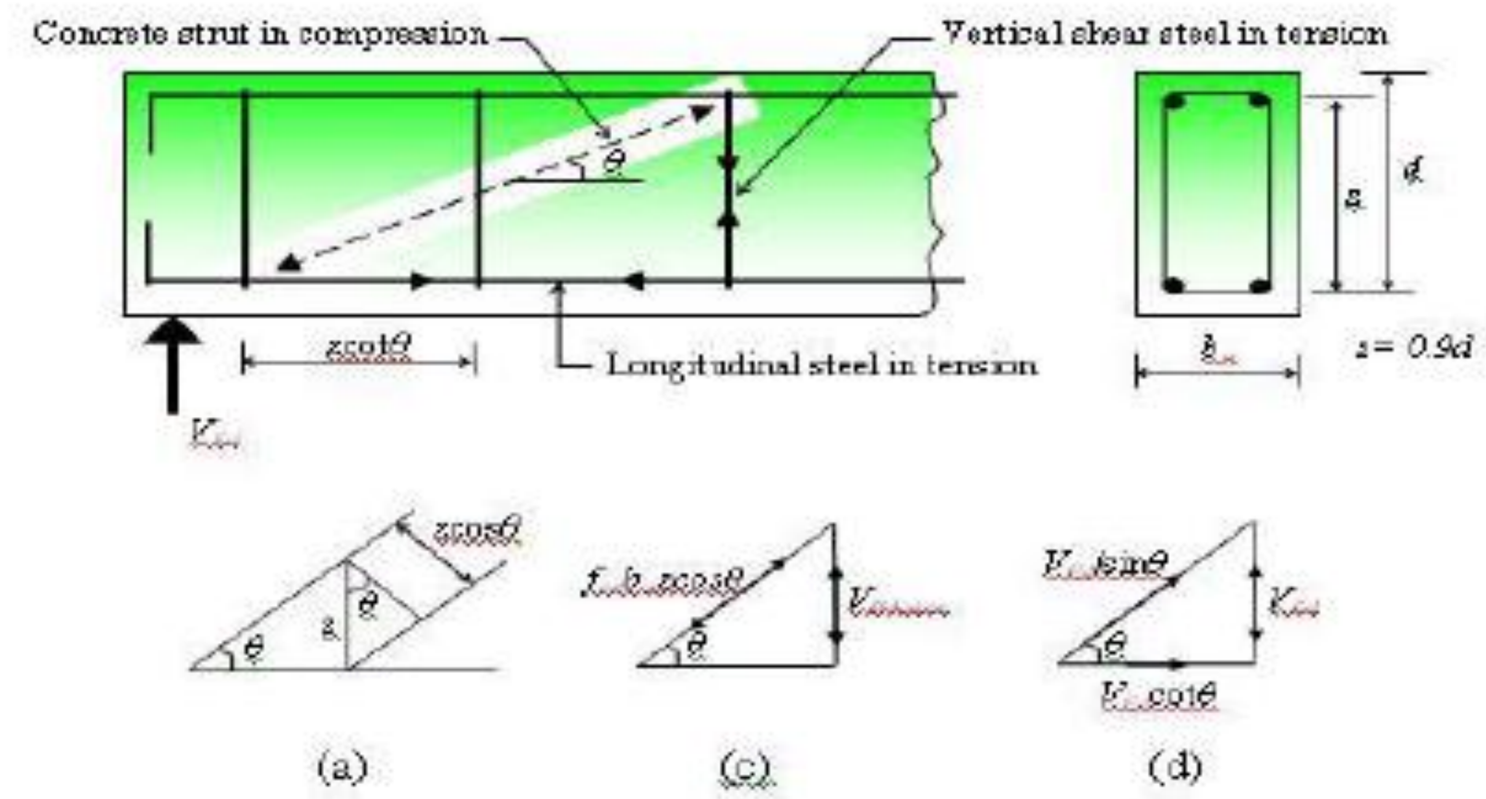


Figure 6.5: Truss model and notation for shear reinforced members

Design for shear

Assumed truss model for the strut inclination method



Design for shear

- The diagonal compressive strut

$$V_{Rd,\ max} = f_{cd} \text{ x } (b_w \text{ x } z \cos\theta) \text{ x } \sin\theta$$
$$= f_{cd}\, b_w\, z \cos\theta \sin\theta$$

- In EC2 this equation is modified by the inclusion of a strength reduction factor for concrete cracked in shear v_1 and the introduction of coefficient taking account of the state of the stress in compression chord α_{cw} thus,

$$V_{Rd,\ max} = \alpha_{cw} v_1 f_{cd}\, b_w\, z / (\cot\theta + \tan\theta)$$
$$= \alpha_{cw} v_1 (f_{ck}/1.5)\, b_w\, 0.9d / (\cot\theta + \tan\theta)$$
$$= \alpha_{cw} v_1 0.6 f_{ck}\, b_w d / (\cot\theta + \tan\theta)$$

Design for shear

- It is set by the EC2 to limit the θ value between 22 to 45 degrees. The recommended value for α_{cw} and v_1 are given in Clause 6.2.3 EC2. For the purpose of this module the following values are used, $\alpha_{cw} = 1.0$, $v_1 = 0.6\,(1 - f_{ck}/250)$ hence,

$$V_{Rd,max} = \frac{0.36 f_{ck} b_w d (1 - f_{ck}/250)}{(\cot\theta + \tan\theta)}$$

- The shear resistance of the link is given by;

$$\begin{aligned} V_{Rd,s} &= f_{ywd} A_{sw} \\ &= (f_{yk}/1.15)\, A_{sw} \\ &= 0.87 f_{yk} A_{sw} \end{aligned}$$

Design for shear

- If the links are spaced at a distance s apart, then the shear resistance of the link is increased proportionately and is given by;

$$
\begin{aligned}
V_{Rd,s} &= 0.87 f_{yk} A_{sw} (z \cot \theta / s) \\
&= 0.87 f_{yk} A_{sw} (0.9d \cot \theta / s) \\
&= 0.78 f_{yk} A_{sw} d (\cot \theta / s)
\end{aligned}
$$

- All shear force will be resisted by the provision of links with no direct contribution from shear capacity of concrete itself.

$$
\begin{aligned}
V_{Ed} &= V_{Rd,s} \\
&= 0.78 f_{yk} A_{sw} d (\cot \theta / s)
\end{aligned}
$$

Design for shear

- Thus rearranging

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78 f_{yk} d \cot\theta}$$

- EC2 (Cl. 9.2.2) specifies a minimum value for A_{sw}/s such that,

$$\frac{A_{sw}}{s} = \frac{0.08 b_w \sqrt{f_{ck}}}{f_{yk}}$$

- EC2 (Cl. 9.2.2) also specifies that the maximum spacing of vertical link should not exceed 0.75*d*.

Design for shear

Design procedures;

- Determine design shear force V_{ed}
- Determine the concrete strut capacity for cot θ *=1.0 and cot* θ *=2.5 (*θ *= 22º and* θ *= 45º respectively)*

$$V_{Rdmax} = \frac{0.36 b_w d f_{ck}}{(\cot\theta + \tan\theta)}$$

- If $V_{Ed} > V_{Rd,max}$ *cot* θ = 1.0 Redesign section
- If $V_{Ed} < V_{Rd,max}$ *cot* θ = 2.5, use cot θ = 2.5, and calculate the shear reinforcement as follows

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78 f_{yk} d \cot\theta} \qquad (\cot\theta = 2.5)$$

Design for shear

- If $V_{Rd,max} \cot\theta = 2.5 < V_{Ed} < V_{Rd,max} \cot\theta = 1.0$
  - Calculate $\theta = 0.5\sin^{-1}\left[\dfrac{V_{Ed}}{0.18 b_w d f_{ck}(1 - f_{ck}/250)}\right]$
  - Calculate shear link as

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78 f_{yk} d \cot\theta}$$

- Calculate the minimum links required by EC2: Cl 9.2.2(5),

$$\frac{A_{sw}}{s} = \frac{0.08 b_w \sqrt{f_{ck}}}{f_{yk}}$$

Design for shear

- Calculate the additional longitudinal tensile force caused by the shear

$$\Delta F_{td} = 0.5 V_{Ed} \cot \theta$$

- Determine additional tension reinforcement,

$$A_s = \Delta f_{td} / 0.87 f_{yk}$$

Deflection

Tables 7.4N

To control deflection to a maximum of span/250:

i. Calculate $\rho_o = \sqrt{f_{ck}}\, 10^{-3}$

ii. Calculate $\rho = A_{s,req} / bd$

iii. Calculate $\rho' = A_{s',req} / bd$

iv. Determine K and calculate l/d

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_o}{\rho} - 1\right)^{3/2}\right] \quad \text{if } \rho \leq \rho_o$$

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho}}\right] \quad \text{if } \rho > \rho_o.$$

Deflection

Calculate Modification factors:

i. Modification factor for span greater than 7m = 7/L

ii. Modification factor for steel area provided= $A_{s,prov}/A_{s,req}$

$$A_{s,prov}/A_{s,req} < 1.5$$

$$(l/d)allow = l/d \times \text{Modification factor } (n_i)$$

$$(l/d)actual = l_{eff}/d_{eff}$$

$$(l/d)allow > (l/d)actual$$

Cracking

Cl.7.3, Tables 7.2N,7.3N

i. Calculate steel stress for limiting crack width, w_{max}=0.3mm

$$f_s = \frac{f_{yk}}{1.15}\left[\frac{G_k + 0.3Q_k}{1.35G_k + 1.5Q_k}\right]\frac{1}{\delta}$$

Or

$$\mathbf{f_s = 435 \times (G_k + 0.3\times Q_k /1.35\times G_k + 1.5\times Q_k) \times (A_{s\,req} / A_{s\,prov})}$$

ii. Calculate bar spacing

Detailing

i. Check curtailment , Cl.9.3.1.1(4), 9.2.1.3, Fig.9.2

ii. Check anchorage, Cl.9.3.1.2, 8.4.4, 9.3.1.19(4), 9.2.1.5(1), 9.2.1.5(2)

iii. Check laps, Cl.8.7.3