ANALYSIS OF SECTION

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Behaviour of Beam in Bending

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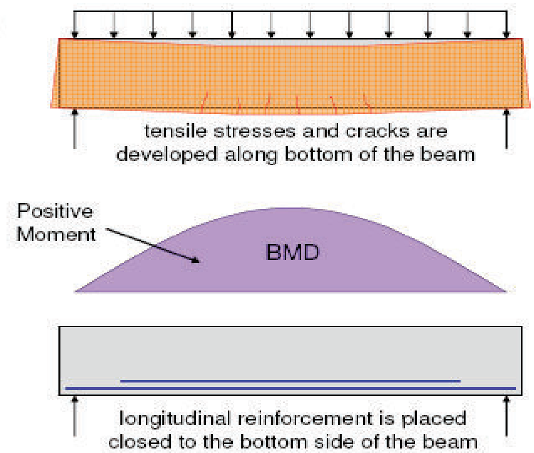
- Consider a simply supported beam subjected to gradually increasing load. The load causes the beam to bend and exert a bending moment as shown in figure below.

- The top surface of the beam is seen to shorten under compression, and the bottom surface lengthens under tension.
- As the concrete cannot resist tension, steel reinforcement is introduces at the bottom surface to resist tension.



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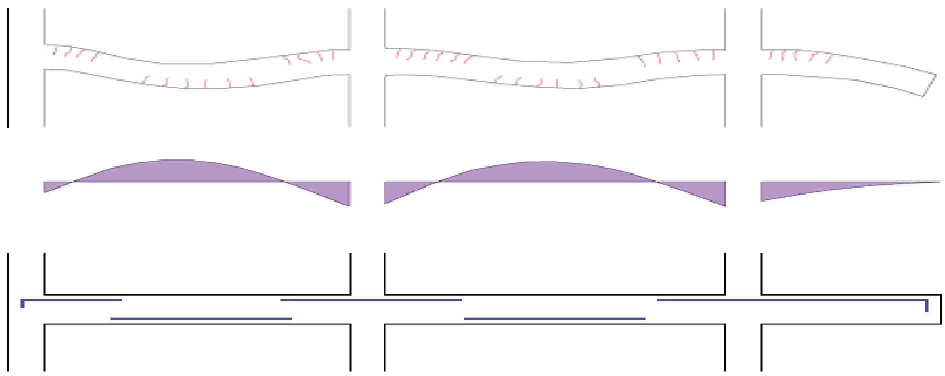
Behavior of Beam in Bending

- For continuous beam, the loads also cause the to bend downward between the support and upward bending over the support.
- This will produce tensile zone as shown in figure below. As the concrete cannot resist flexural tension, steel reinforcement would be introduced as detail in the figure.



Basic Assumption in RC Design

- In the design of reinforced concrete beam the following **assumptions** are made (See EN 1991: Cl. 6.1 (2) P.)
  - Plane section through the beam before bending remain plane after bending.
  - The strain in bonded reinforcement, whether in tension or compression is the same as that in the surrounding concrete.
  - The tensile of the concrete is ignored.
  - The stresses in the concrete and reinforcement can be derived from the strain by using stress-strain curve for concrete and steel.

Basic Assumption in RC Design

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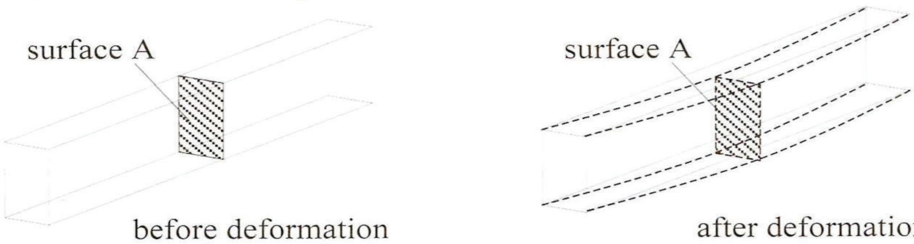
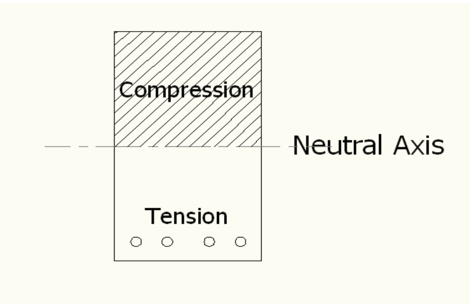
- Plane sections remain plane, i.e.



The surface of any cross-section does **not** distort out-of-plane during deformation.



Distribution of Stresses and Strain

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- Figure below shows the cross section of a RC beam subjected to bending and the resultant strain and stress distribution in the concrete.
- Top surface of cross section are subjected to compressive stresses while the bottom surface subjected to tensile stresses.
- The line that introduced in between the tensile and compression zones is known as the neutral axis of the member.
- Due to the tensile strength of concrete is very low, all the tensile stresses at the bottom fibre are taken by reinforcement.

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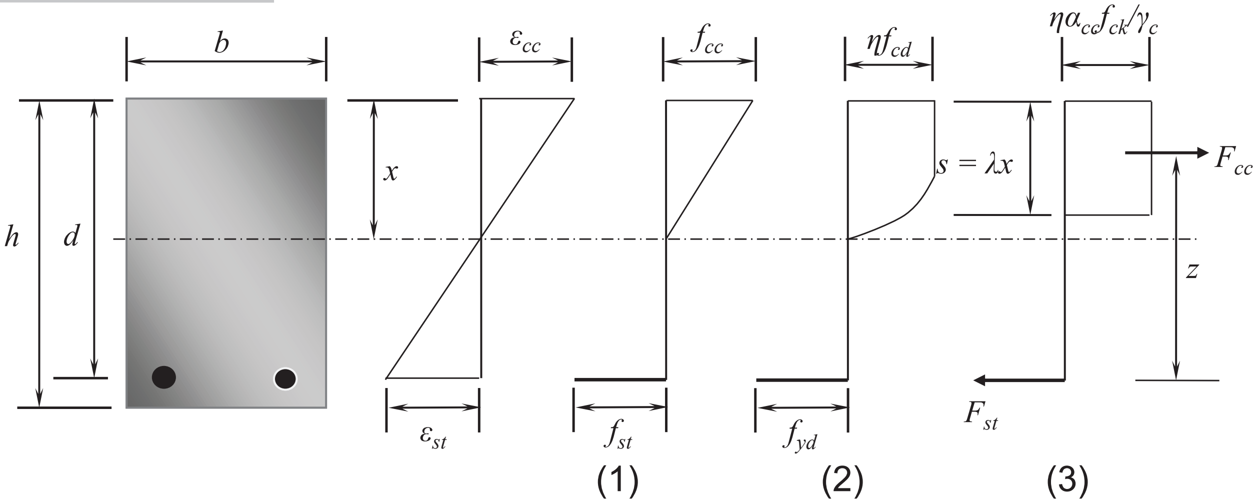
Distribution of Stresses and Strain



For f_{ck} < 50 N/mm²: η = 1 (defining the effective strength), ε_c = 0.0035,

α_{cc} = 0.85, λ = 0.8, γ_c = 1.5,

f_{cd} = 1.0 x 0.85 x f_{ck} / 1.5 = 0.567 f_{ck}

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Distribution of Stresses and Strain

Stress distribution in the concrete

1. The triangular stress distribution applies when the stress are very nearly proportional to the strain, which generally occurs at the loading levels encountered under working load conditions and is, therefore, used at the serviceability limit state.
2. The rectangular-parabolic stress block represents the distribution at failure when the compressive strain are within the plastic range, and it is associated with the design for ultimate limit state.
3. The equivalent rectangular stress block is a simplified alternative to the rectangular-parabolic distribution.

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Distribution of Stresses and Strain

- The distribution of strains across the beam cross section is linear. That is, the normal strain at any points in a beam section is proportional to its distance from the neutral axis.
- The steel strain in tension ε_{st} can be determined from the strain diagram as follows:

$$\frac{\varepsilon_{st}}{(d-x)} = \frac{\varepsilon_{cc}}{x} \Rightarrow \varepsilon_{st} = \varepsilon_{cc}\left(\frac{d-x}{x}\right)$$

Therefore ;

$$x = \frac{d}{1+\left(\frac{\varepsilon_{st}}{\varepsilon_{cc}}\right)}$$

Introduction

- Since, ε_{cc} = 0.0035 for class ≤ C50/60 and
- For steel with f_{yk} = 500 N/mm^2 and the yield strain is ε_{st} = 0.00217.
- By substituting ε_{cc} and ε_{st} ,

$$x = 0.617d$$

- Hence, to ensure yielding of the tension steel at limit state the depth of neutral axis, x should be less than or equal to $0.617d$.

Type of RC Beam Failure

- As applied moment on the beam section increased beyond the linear elastic stage, the concrete strains and stresses enter the nonlinear stage.
- The behavior of the beam in the nonlinear stage depends on the amount of reinforcement provided.
- The reinforcing steel can sustain very high tensile strain however, the concrete can accommodate compressive strain much lower compare to it.
- So, the final collapse of a normal beam at ultimate limit state is cause by the crushing of concrete in compression, regardless of whether the tension steel has yield or not.

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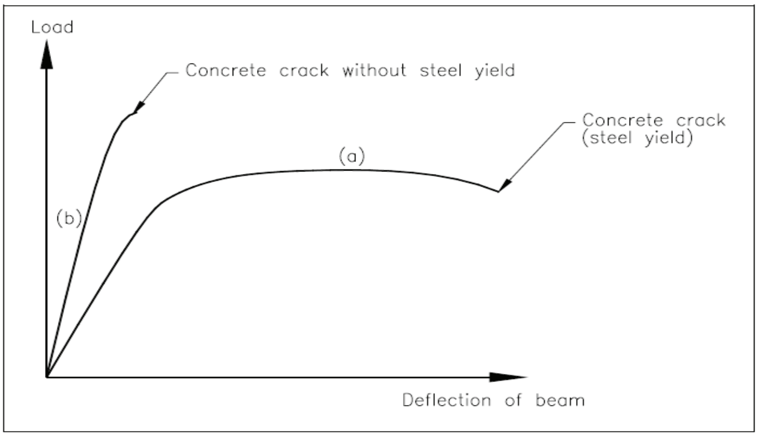
Type of RC Beam Failure

- Depending on the amount of reinforcing steel provided, flexural failure may occur in three ways:
  - **Balanced** : Concrete crushed and steel yields simultaneously at the ultimate limit state. The compressive strain of concrete reaches the ultimate strains ε_{cu} and the tensile strain of steel reaches the yield strain ε_y simultaneously. The depth of neutral axis, **x = 0.617d**.
  - **Under-reinforced** : Steel reinforcement yields before concrete crushes. The area of tension steel provided is less than balance section. The depth of neutral axis, **x < 0.617d**. The failure is gradual, giving ample prior warning of the impending collapse. This mode if failure is preferred in design practice.
  - **Over-reinforced** : Concrete fails in compression before steel yields. The area of steel provided is more than area provided in balance section. The depth of neutral axis, **x > 0.617d**. The failure is sudden (without any sign of warning) and brittle. Over-reinforced are not permitted.

With Wisdom We Explore

Introduction

For a singly reinforced beam EC2 limits the depth to the neutral axis, x to $0.45d$ $(x \leq 0.45d)$ for concrete class ≤ *C50/60* to ensure that the design is for the under-reinforced case where failure is gradual, as noted above. For further understanding, see the graph shown below .



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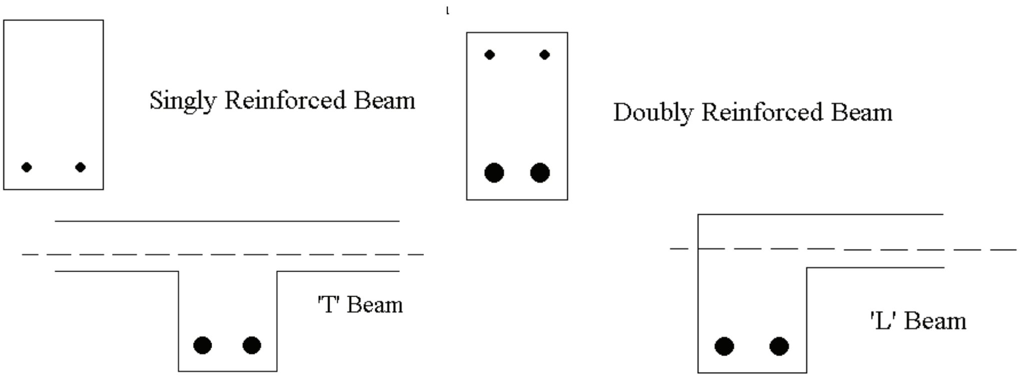
Analysis of Section

- Section 6.1 EN 1992-1-1, deal with the analysis and design of section for the ultimate limit state design consideration of structural elements subjected to bending.
- The two common types of reinforced concrete beam section are:
  - Rectangular section : Singly and doubly reinforced
  - Flanged section : Singly and doubly reinforced



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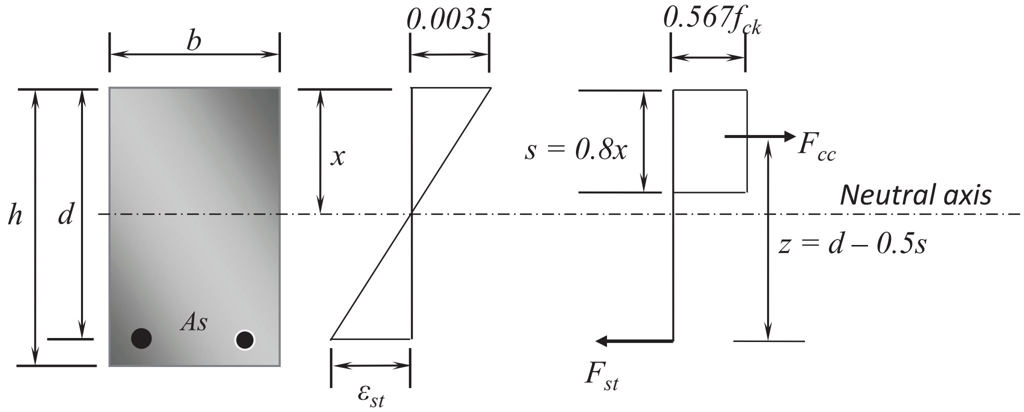
- Beam cross section, strains and stresses distribution at ULS of singly reinforced rectangular beam



Notation:

h	=	Overall depth	d	=	Effective depth
b	=	Width of section	s	=	Depth of stress block
A_s	=	Area of tension reinforcement	x	=	Neutral axis depth
f_{ck}	=	Characteristic strength of concrete	z	=	Lever arm
f_{yk}	=	Characteristic strength of reinforcement			

Tension force of steel, $\boldsymbol{F_{st}}$

$$F_{st} = Stress\ x\ Area = 0.87 f_{yk} \times A_s$$

Compression force of concrete, $\boldsymbol{F_{cc}}$

$$F_{cc} = Stress\ x\ Area = 0.567 f_{ck}(b \times 0.8x) = 0.454 f_{ck} bx$$

For equilibrium, total force in the section should be zero.

$$F_{cc} = F_{st}$$

$$0.454 f_{ck} bx = 0.87 f_{yk} A_s$$

$$x = \frac{0.87 f_{yk} A_s}{0.454 f_{ck} b}$$

Singly Reinforced Rectangular Beam

Moment resistance with respect to the steel

$$M = F_{cc} \times z$$

$$M = (0.454 f_{ck} bx)(d - 0.4x) = \left(\frac{0.454x}{d}\right)\left(\frac{d-0.4x}{d}\right)(f_{ck}.b.d^2)$$

Lets; $\left(\frac{0.454x}{d}\right)\left(1-\frac{0.4x}{d}\right) = K$ Therefore; $M = K.f_{ck}.b.d^2$

Moment resistance with respect to the concrete

$$M = F_{st} \times z$$

$$M = (0.87 f_{yk} A_s)(d - 0.4x)$$

Area of tension reinforcement, $A_s = \dfrac{M}{0.87.f_{yk}.(d-0.4x)}$

Singly Reinforced Rectangular Beam

To ensure that the section designed is under-reinforced it is necessary to place a limit on the maximum depth of the neutral axis (x). EC2 suggests:

$$x \leq 0.45d$$

Then ultimate moment resistance of singly reinforced section or M_{bal} can be obtained by;

$$M_{bal} = (0.454 f_{ck} bx)(d - 0.4x)$$

$$M_{bal} = [0.454.f_{ck}.b(0.45d)].[d - 0.4(0.45d)]$$

$$M_{bal} = (0.2043.f_{ck}.b.d).(0.82d)$$

$$M_{bal} = 0.167.f_{ck}.b.d^2 \ @ \ K_{bal}.f_{ck}.b.d^2$$

Singly Reinforced Rectangular Beam

Therefore;

- $M \quad = K.f_{ck}.b.d^2$
- $M_{bal} = K_{bal}.f_{ck}.b.d^2$

  where; $K_{bal} = 0.167$

If;

- $M \leq M_{bal}$ or $K \leq K_{bal}$: Singly reinforced rectangular beam
  (Tension reinforcement only)
- $M > M_{bal}$ or $K > K_{bal}$: Doubly reinforced rectangular beam
  (Section requires compression reinforcement)

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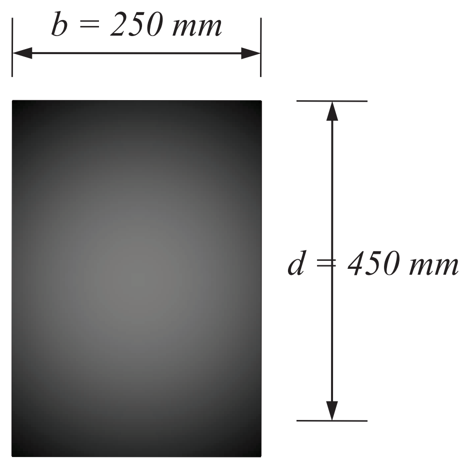
Example 3.1

The cross section of rectangular beam is shown in figure below. Using stress block diagram and the data given, determine the area and the number of reinforcement required.

Data:

Design moment, M_{ED}	= 200 kN.m
f_{ck}	= 25 N/mm^2
f_{yk}	= 500 N/mm^2



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Solution of Example 3.1

Calculate the ultimate moment resistance of section, M_{bal}

$$M_{bal} = 0.167.f_{ck}.b.d^2$$
$$= 0.167(25)(250)(450^2)$$
$$= 211.36kNm > M = 200kNm$$

Singly reinforced section

Neutral axis depth, x

$$M = (0.454 f_{ck} bx)(d - 0.4x)$$

$$200 \times 10^6 = 0.454(25)(250)(x)(450 - 0.4x)$$

$$x^2 - 1125x + 176211.45 = 0$$

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Solution of Example 3.1

$x = 188\ mm\ @\ 937\ mm$

Use $x = 188\ mm$

Checking; $\quad \dfrac{x}{d} = \dfrac{188}{450} = 0.42 < 0.45$

Lever arm, $z = (d - 0.4x)$

$= (450 - 0.4(188)) = 374.8\ mm$

Area of reinforcement, A_s

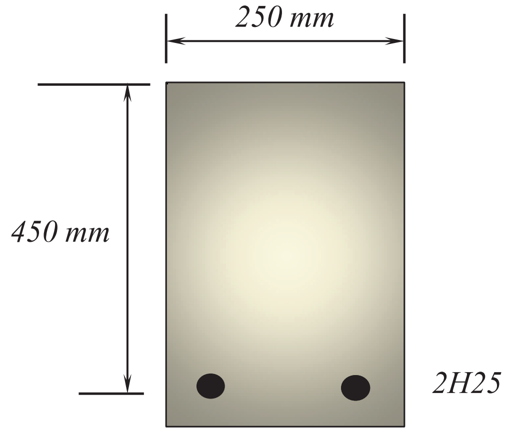
$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{200 \times 10^6}{0.87(500)(374.8)} = 1227mm^2$$

$$\text{Provide } 4H20\ (A_{sprov} = 1257mm^2)$$

With Wisdom We Explore

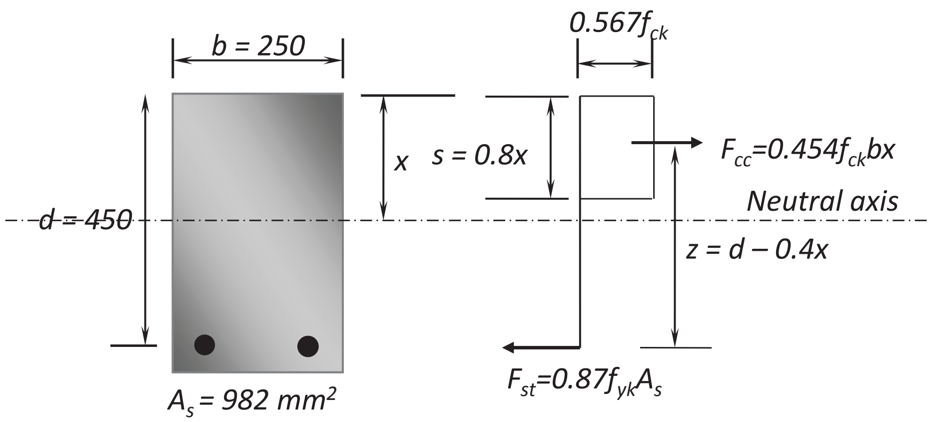
Example 3.2

Figure below shows the cross section of a singly reinforced beam. Determine the resistance moment for that cross section with the assistance of a stress block diagram. Given f_{ck}= 25 N/mm^2 and f_{yk} = 500 N/mm^2.



Solution of Example 3.2

A stress block diagram is drawn with the important values and notations.



For equilibrium;

$$F_{cc} = F_{st}$$

$$0.454 f_{ck} bx = 0.87 f_{yk} A_s$$

$$x = \frac{0.87 f_{yk} A_s}{0.454 f_{ck} b}$$

Solution of Example 3.2

$$x = \frac{0.87(500)(982)}{0.454(25)(250)} = 151\,mm$$

Checking;

$$\frac{x}{d} = \frac{151}{450} = 0.34 < 0.45$$

Moment resistance of section;

$$M = F_{cc} \times z \qquad @ \qquad M = F_{st} \times z$$

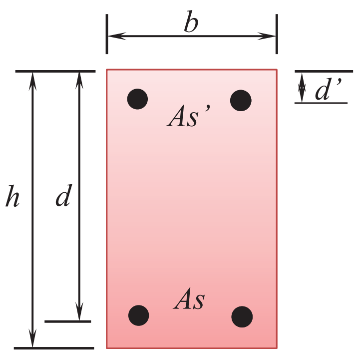
$$M = (0.454 f_{ck} bx)(d - 0.4x)$$

$$M = (0.454 \times 25 \times 250 \times 151)(450 - 0.4(151))$$

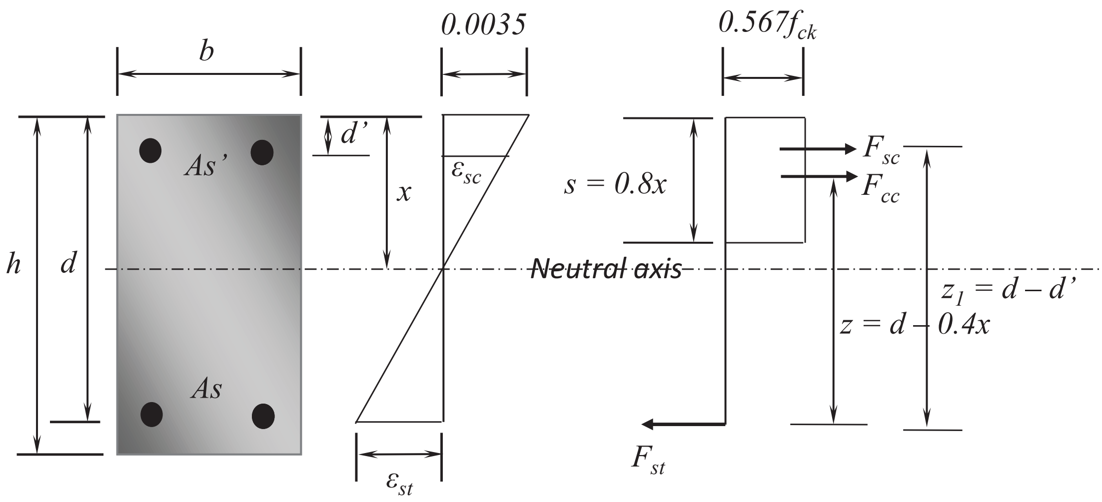
$$M = 167\ kNm$$

Doubly Reinforced Rectangular Beam

- When the load applied increases gradually and it will reach a state that the compressive strength of concrete is not adequate to take additional compressive stress.
- Compression reinforcement is required to take the additional compressive stress.
- This section is named as doubly reinforced section.



Strain and stress block diagrams of doubly reinforced beam.



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Doubly Reinforced Rectangular Beam

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Internal force;

$$F_{cc} = 0.454 f_{ck} bx$$

$$F_{st} = 0.87 f_{yk} A_s \qquad \text{and} \qquad F_{sc} = 0.87 f_{yk} A_s'$$

Lever arms;

$$z = d - 0.4x \qquad z_1 = d - d'$$

For equilibrium of internal force;

$$F_{st} = F_{cc} + F_{sc}$$

$$0.87 f_{yk} A_s = 0.454 f_{ck} bx + 0.87 f_{yk} A_s'$$

With Wisdom We Explore

Doubly Reinforced Rectangular Beam

Taking moment about the centroid of the tension steel,

$$M = F_{cc}.z + F_{sc}.z_1$$
$$M = (0.454 f_{ck} bx).(d - 0.4x) + (0.87 f_{yk} A_s').(d - d')$$

For design purpose, $x = 0.45d$

$$M = (0.454 f_{ck} bx).[d - 0.4(0.45d)] + (0.87 f_{yk} A_s').(d - d')$$
$$= 0.167 f_{ck} bd^2 + (0.87 f_{yk} A_s').(d - d')$$
$$= M_{bal} + (0.87 f_{yk} A_s').(d - d')$$

The area of compression reinforcement, A_s'

$$A_s' = \frac{(M - M_{bal})}{0.87 f_{yk} (d - d')} \quad \text{or} \quad A_s' = \frac{(K - K_{bal}) f_{ck} bd^2}{0.87 f_{yk} (d - d')}$$

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Doubly Reinforced Rectangular Beam

The area of Tension reinforcement, A_s

Multiplied equilibrium internal force equation by z ,

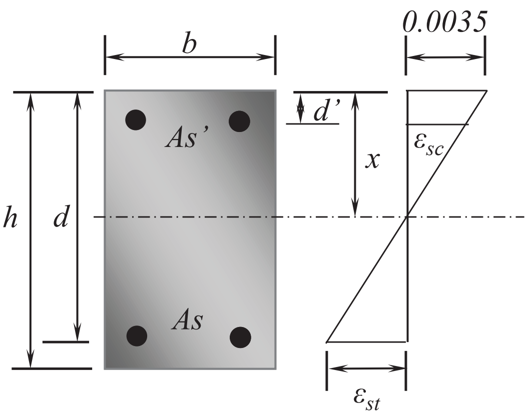
Limiting $x = 0.45d$ and $z = d - 0.4(0.45d) = 0.82d$

$$0.87 f_{yk} A_s z = 0.454 f_{ck} bxz + 0.87 f_{yk} A_s' z$$
$$0.87 f_{yk} A_s z = 0.454 f_{ck} b(0.45d)(0.82d) + 0.87 f_{yk} A_s' z$$
$$0.87 f_{yk} A_s z = 0.167 f_{ck} bd^2 + 0.87 f_{yk} A_s' z$$

$$A_s = \frac{0.167 f_{ck} bd^2}{0.87 f_{yk} z} + A_s' \quad \text{or} \quad A_s = \frac{K_{bal} f_{ck} bd^2}{0.87 f_{yk} z} + A_s'$$

Stress in compression reinforcement.

- The derivation of design formula for doubly reinforced section assumed that the compression reinforcement reaches the design strength of $0.87f_{yk}$ at ultimate limit state.
- From the strain diagram as shown in figure below.



$$\frac{\varepsilon_{sc}}{(x-d')} = \frac{0.0035}{x}$$

$$\frac{(x-d')}{x} = \frac{\varepsilon_{sc}}{0.0035}$$

$$\frac{d'}{x} = 1 - \left(\frac{\varepsilon_{sc}}{0.0035}\right)$$

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Doubly Reinforced Rectangular Beam

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For the design strength $0.87f_{yk}$ to be reached, $\varepsilon_{sc} = 0.87f_{yk} / E_s$

$$\varepsilon_{sc} = \frac{0.87 f_{yk}}{E_s} = \frac{0.87(500)}{200\times 10^3} = 0.002175$$

$$\frac{d'}{x} = 1 - \left(\frac{0.002175}{0.0035}\right) = 0.38$$

Therefore, if $d'/x < 0.38$ the compression reinforcement can be assumed reach the design strength of $0.87f_{yk}$. If $d'/x > 0.38$, a reduced stress should be used.

$$f_{sc} = E_s . \varepsilon_{sc}$$

$$f_{sc} = 200\times 10^3 (0.0035)(1 - d'/x) \quad = 700(1 - d'/x)$$

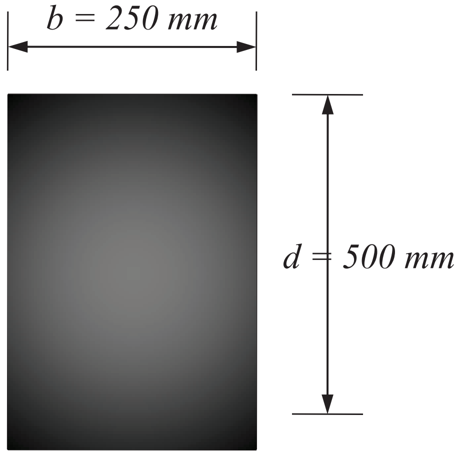
With Wisdom We Explore

Example 3.3

The cross section of rectangular beam is shown in figure below. Using the data given, determine the area and the number of reinforcement required.

<u>**Data:**</u>

Design moment, M_{ED}	= 450 kN.m
f_{ck}	= 25 N/mm^2
f_{yk}	= 500 N/mm^2
d’	= 50 mm



Solution of Example 3.3

Ultimate moment resistant of section, M_{bal}

$$M_{bal} = 0.167.f_{ck}.b.d^2$$
$$= 0.167(25)(250)(500^2)(10^{-6})$$
$$= 260.94kNm < M = 450kNm$$

Compression reinforcement is required

Area of compression reinforcement, A_s’

$$A_s' = (M - M_{bal}) / 0.87 f_{yk}(d - d')$$
$$= (450 - 260.94) \times 10^6 / 0.87(500)(500 - 50)$$
$$= 966mm^2$$

Solution of Example 3.3

Checking d'/x ratio

$$x = 0.45d = 0.45(500) = 225mm$$

$$d'/x = 50/225 = 0.22 < 0.38$$

Compression steel achieved it design strength at $0.87f_{yk}$

Area of tension steel, A_s

$$A_s = \left(\frac{M_{bal}}{0.87 f_{yk} z}\right) + A_s' = \frac{260.94 \times 10^6}{0.87 \times 500 \times (0.82 \times 500)} + 966$$

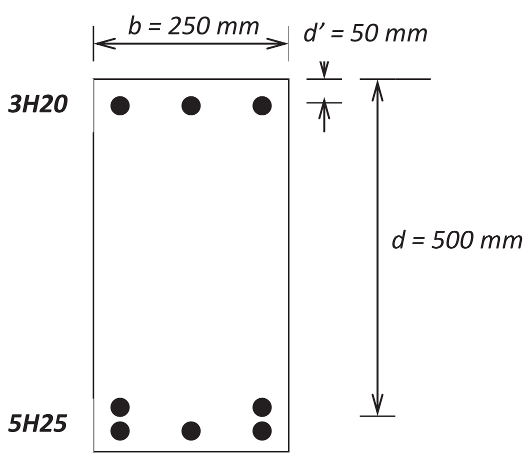
$$= 2429mm^2$$

Provide 2H25 ($A_{s'\,Prov.}$ = 982 mm^2) – Compression reinforcement

5H25 ($A_{s\,Prov.}$ = 2454 mm^2) – Tension reinforcement

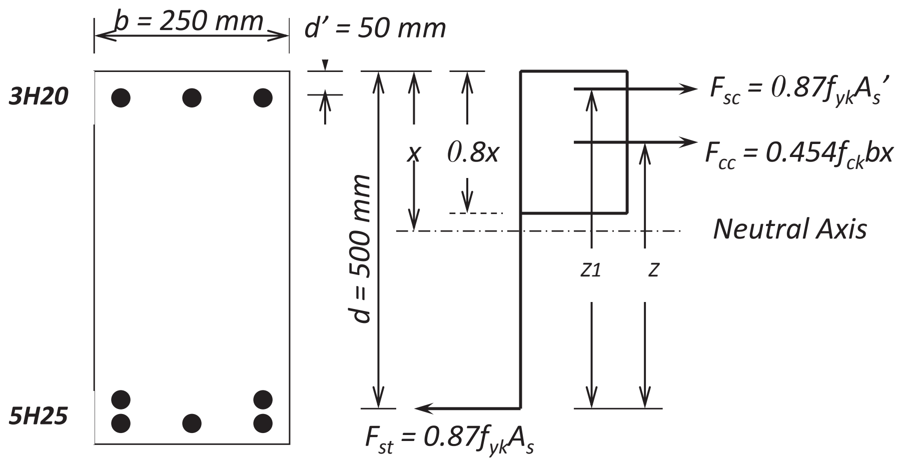
Example 3.4

Calculate moment resistance of the doubly reinforced section shown in figure below. Given f_{ck} = 30 N/mm^2 and f_{yk} = 500 N/m^2.



Solution of Example 3.4

A stress block diagram is drawn with the important values and notations



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Solution of Example 3.4

Reinforcement used 3H20, $A_{s'} = 943\ mm^2$ & 5H25, $A_s = 2455\ mm^2$

Neutral axis depth, x

$$x = \frac{0.87 f_{yk}(A_s - A_s')}{0.454 f_{ck} b} = \frac{0.87(500)(2455-943)}{0.454(30)(250)}$$

$$x = 193mm$$

Checking the stress of steel

$$x/d = 193/500 = 0.39 < 0.45$$

$$d'/x = 50/193 = 0.26 < 0.38$$

Steel achieved it design strength $0.87f_y$ as assumed

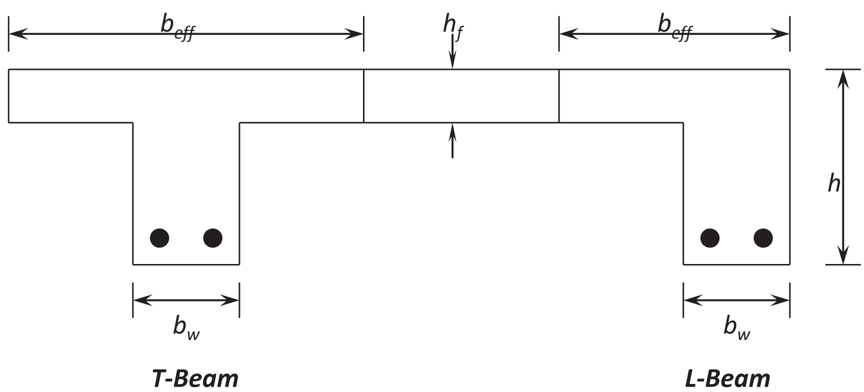
Solution of Example 3.4

Moment resistance of section, M

$$M = F_{sc}.z_1 + F_{cc}.z$$
$$= 0.87 f_{yk} A_s'(d - d') + 0.454 f_{ck} bx(d - 0.4x)$$
$$= 0.87(500)(943)(500 - 50)$$
$$+ 0.454(30)(250)(193)(500 - 0.4(193)) \times 10^{-6}$$
$$= 462 kNm$$

Flange Beam

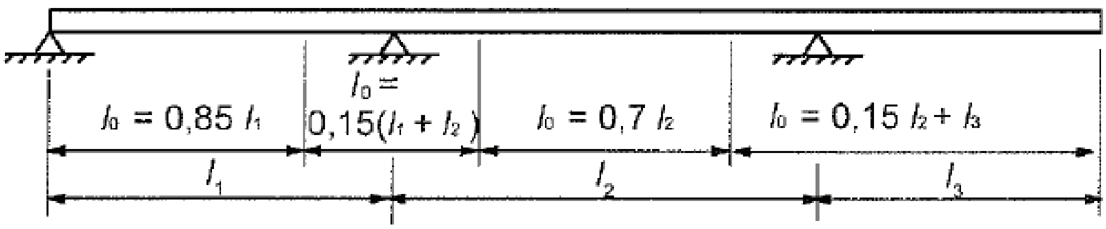
- Flanged beams occur when beams are cast integrally with and support a continuous floor slab.
- Part of the slab adjacent to the beam is counted as acting in compression to form T- and L-beams as shown in figure below.



Where;
b_{eff} = effective flange width
b_w = breadth of the web of the beam.
h_f = thickness of the flange.

Flange Beam

- The effective width of flange, b_{eff} is given in Sec. 5.3.2.1 of EC2.
- b_{eff} should be based on the distance l_o between points of zero moment as shown in figure below.



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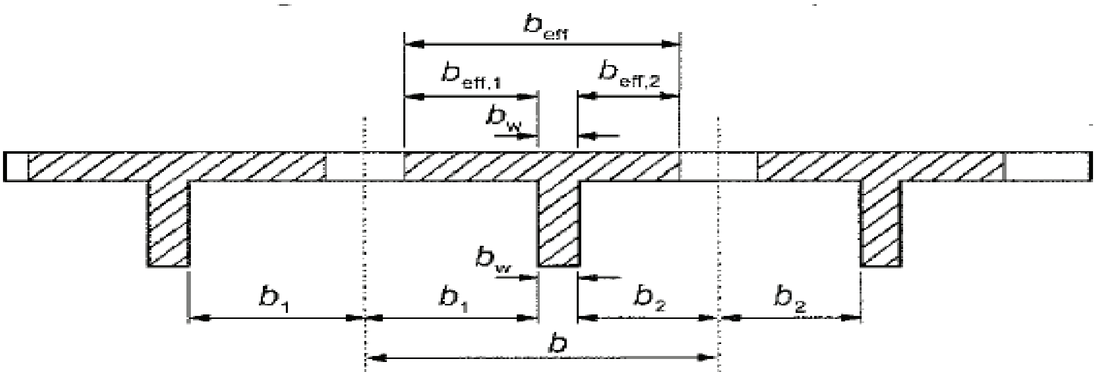
Flange Beam

- The effective flange width, b_{eff} for T-beam or L-beam may be derived as:

$$b_{eff} = \Sigma b_{eff,i} + b_w \leq b$$

- Where

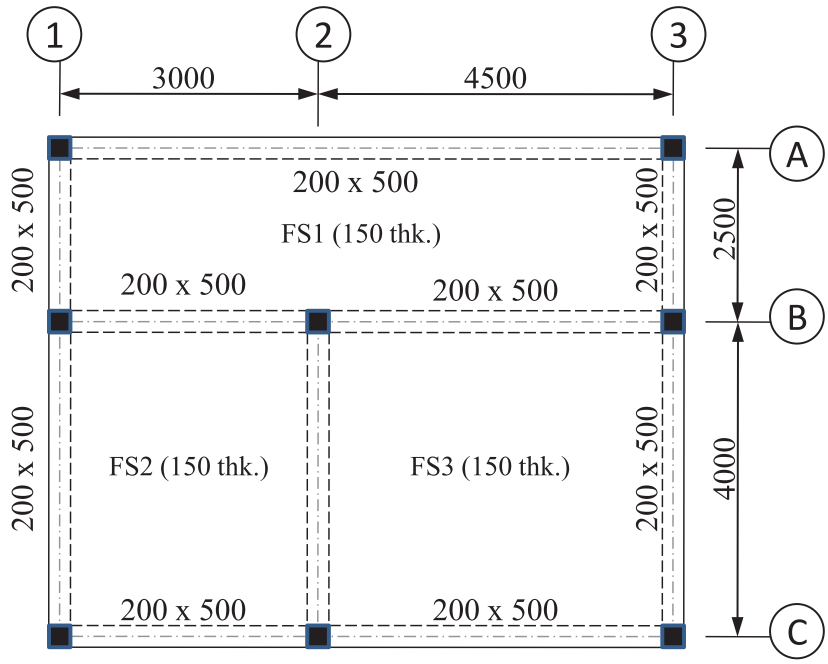
$$b_{eff,i} = 0.2 b_i + 0.1 l_o \leq 0.2 l_o \text{ and } b_{eff,i} \leq b_i$$



With Wisdom We Explore

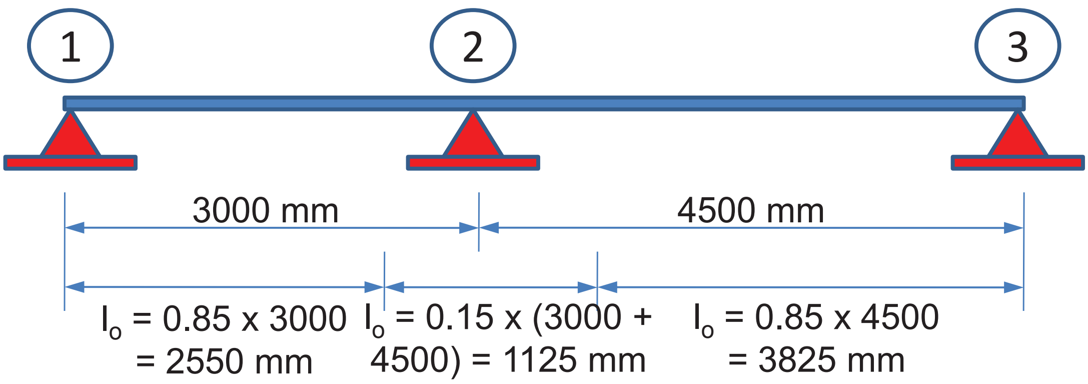
Example 3.5

Based on figure below, determine the effective flange width, b_{eff} of beam B/1-3.



Solution of Example 3.5

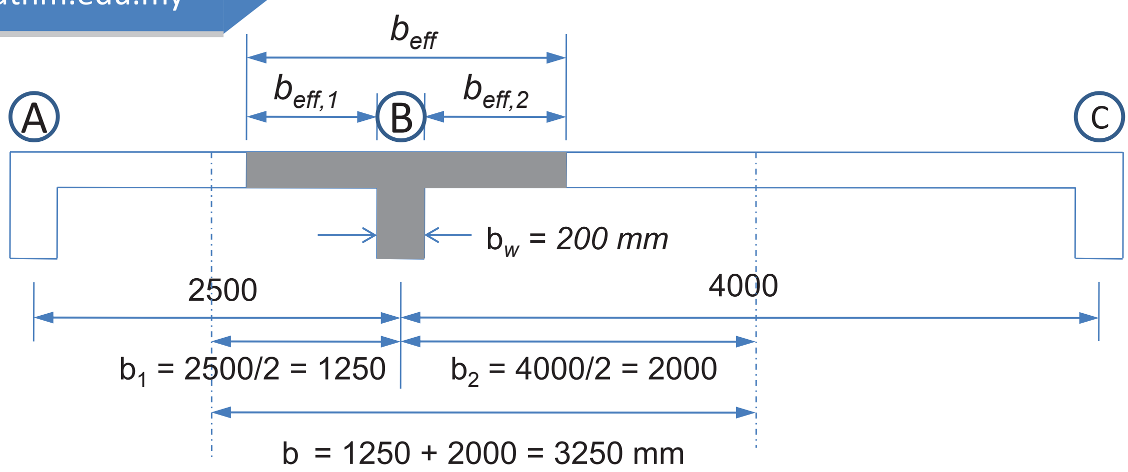
l_o (distance between points of zero moment)



Effective flange width, b_{eff}

$$b_{eff} = \Sigma b_{eff,i} + b_w \leq b$$

Solution of Example 3.5



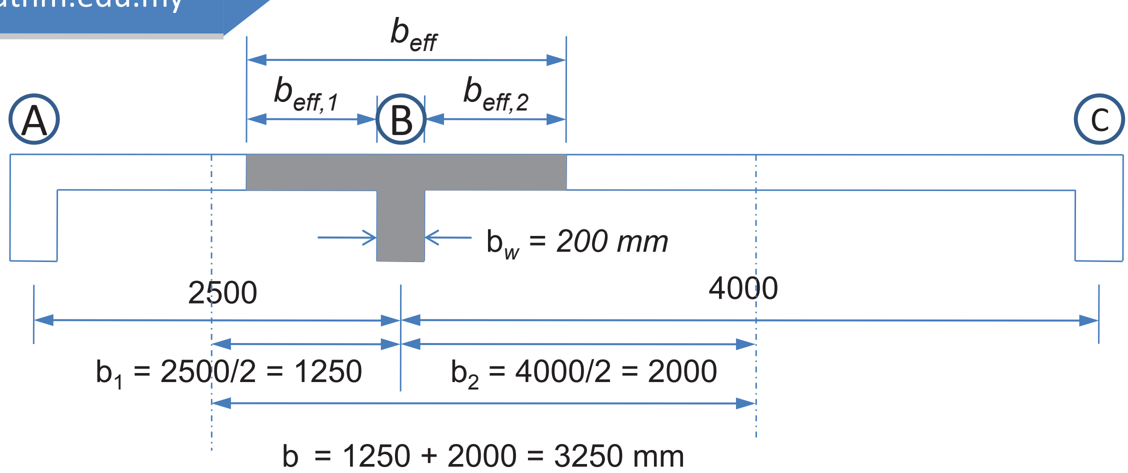
Span 1-2

b_{eff1} = 0.2(1250) + 0.1(2550) = **505 mm** < $0.2l_o$ = 510 mm < b_1 = 1250 mm

b_{eff2} = 0.2(2000) + 0.1(2550) = **655 mm** > $0.2l_o$ = **510 mm** < b_2 = 2000 mm

b_{eff} = (505 + 510) + 200 = **1215 mm** < 3250 mm

Solution of Example 3.5



Span 2-3

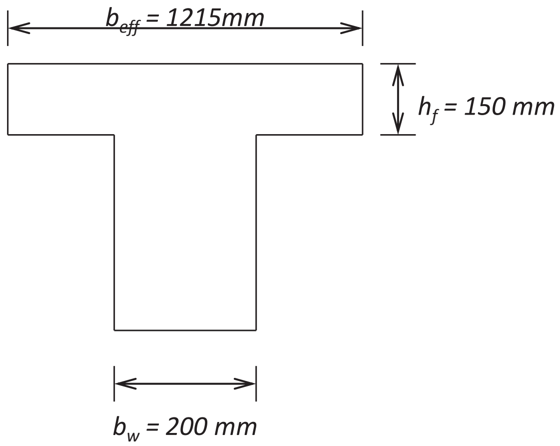
b_{eff1} = 0.2(1250) + 0.1(3825) = **632.5 mm** < $0.2l_o$ = 765 mm < b_1 = 1250 mm

b_{eff2} = 0.2(2000) + 0.1(3825) = **782.5 mm** > $0.2l_o$ = **765 mm** < b_2 = 2000 mm

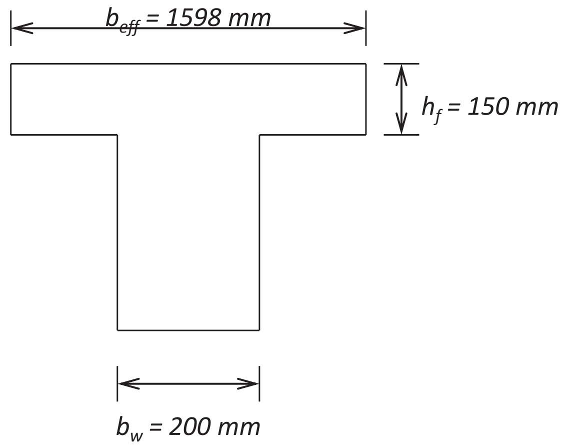
b_{eff} = (632.5 + 765) + 200 = **1597.5 mm** < 3250 mm

Solution of Example 3.5

Span 1-2



Span 2-3



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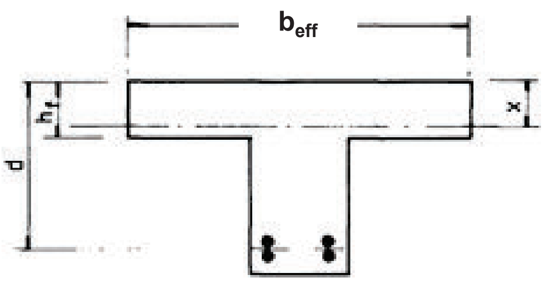
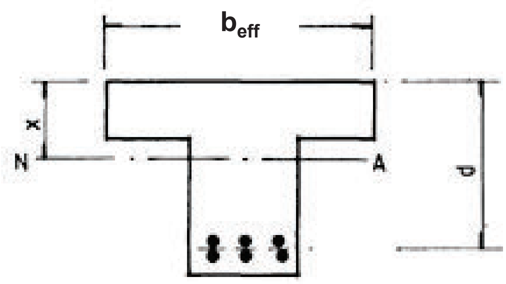
Flange Beam

- The design procedure of flange beam depends on where the neutral axis lies. The neutral axis may lie in the flange or in the web.
- In other word, there are three cases that should be considered.
  - Neutral axis lies in flange $(M < M_f)$
  - Neutral axis lies in web $(M > M_f \text{ but } < M_{bal})$
  - Neutral axis lies in web $(M > M_{bal})$

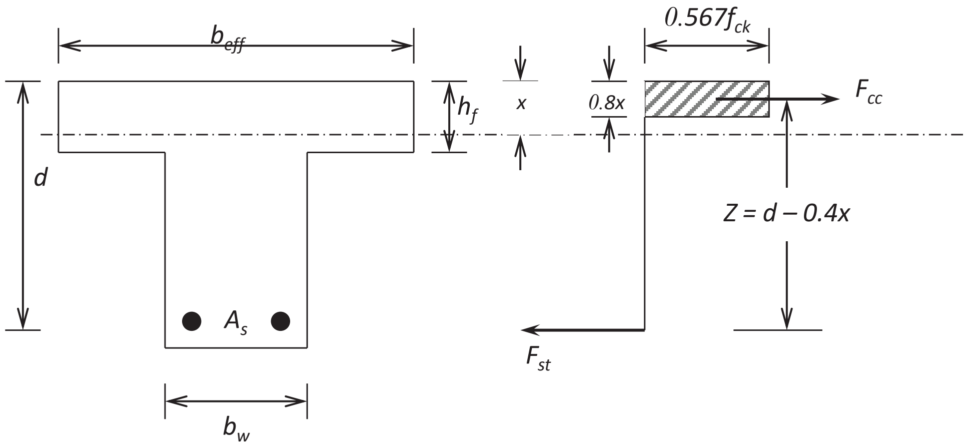




With Wisdom We Explore

Neutral axis lies in flange $(M < M_f)$

- This condition occur when the depth of stress block *(0.8 x)* less then the thickness of flange, h_f as shown in figure below.



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- Moment resistance of section, M

$$M = F_{cc} \times z$$

$$M = (0.567 f_{cu} b_{eff} 0.8x)(d - 0.4x)$$

- For this case, maximum depth of stress block, *0.8x* are equal to *hf*

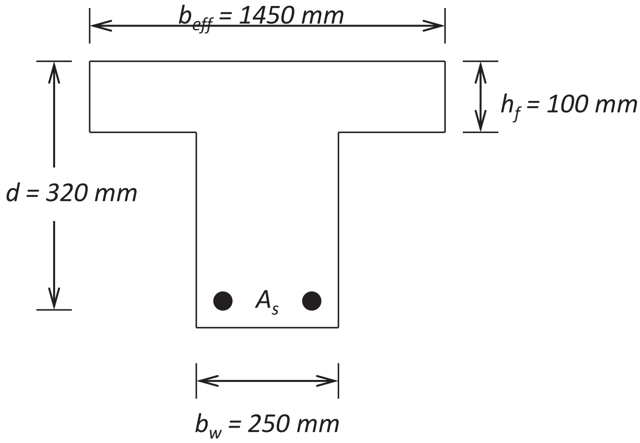
$$M = M_f = (0.567 f_{ck} b h_f)(d - h_f / 2)$$

- Where, M_f = Ultimate moment resistance of flange.
- Therefore, if $M \leq M_f$ the neutral axis lies in flange and the design can be treated as rectangular singly reinforced beam.

$$A_s = \frac{M}{0.87 f_{yk} z} \quad \text{or} \quad A_s = \frac{M}{0.87 f_{yk} (d - 0.4x)}$$

Example 3.6

The T-beam with dimension as shown in figure below is subjected to design moment, $M = 250\ kNm$. If $f_{ck} = 30\ N/mm^2$ and $f_{yk} = 500\ N/mm^2$ have been used, determine the area and number of reinforcement required.



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Solution of Example 3.6

Moment resistance of flange, M_f

$$M_f = (0.567 f_{ck} b_{eff} h_f)(d - h_f/2)$$
$$= (0.567 \times 30 \times 1450 \times 100)(320 - 100/2) \times 10^{-6}$$
$$= 665.9 kNm > M = 250 kNm$$

Since $M < M_f$, Neutral axis lies in flange

Compression reinforcement is not required

$$M = (0.454 f_{ck} bx)(d - 0.4x)$$

$$250 \times 10^6 = 0.454(30)(1450)(x)(320 - 0.4x)$$

$$x^2 - 800x + 31647.2 = 0$$

$$x = 758.3mm @ 41.74mm \quad Use\ x = 41.74mm$$

With Wisdom We Explore

Checking

$$\frac{x}{d} = \frac{41.74}{320} = 0.13 < 0.45$$

Lever arm, z

$$z = d - 0.4x = 320 - 0.45(41.74) = 303.3mm$$

Area of tension reinforcement, A_s

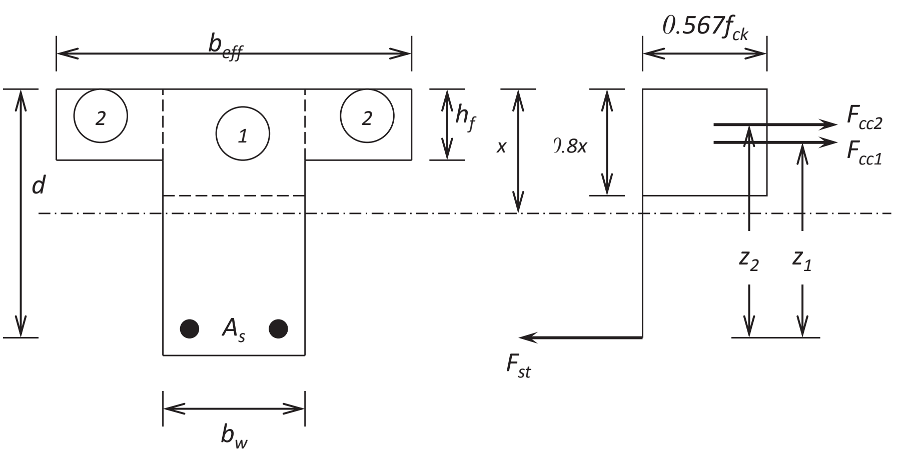
$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{250 \times 10^6}{0.87(500)(303.3)}$$

$$= 1895 mm^2$$

Provide 4H25 (A_{sprov} = 1964 mm²)

Neutral axis lies in web $(M_f < M < M_{bal})$

- If the applied moment M is greater than M_f the neutral axis lies in the web as shown in figure below.



Flange Beam

From the stress block, internal forces;

$$F_{cc1} = (0.567 f_{ck})(b_w.0.8x) = 0.454 f_{ck} b_w x$$

$$F_{cc2} = (0.567 f_{ck})(b_{eff} - b_w) h_f$$

$$F_{st} = 0.87 f_{yk} A_s$$

Lever arms, z

$$z_1 = d - 0.4x \qquad z_2 = d - 0.5h_f$$

Moment resistance, M

$$M = F_{cc1}.z_1 + F_{cc2}.z_2$$
$$= (0.454 f_{ck} b_w x)(d - 0.4x) + (0.567 f_{ck})(b_{eff} - b_w) h_f (d - 0.5h_f)$$

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Flange Beam

Ultimate moment resistance of section, M_{bal} *(When* $x = 0.45d$*)*

$$M_{bal} = (0.454 f_{ck} b_w 0.45d)(d - 0.4(0.45d))$$
$$+ (0.567 f_{ck})(b_{eff} - b_w) h_f (d - 0.5h_f)$$

Divide both side by $f_{ck} b_{eff} d^2$, then;

$$\frac{M_{bal}}{f_{ck} b_{eff} d^2} = 0.167 \frac{b_w}{b_{eff}} + 0.567 \frac{h_f}{d}\left(1 - \frac{b_w}{b_{eff}}\right)\left(1 - \frac{h_f}{2d}\right)$$

$$\frac{M_{bal}}{f_{ck} b_{eff} d^2} = \beta_f$$

Therefore; $M_{bal} = \beta_f f_{ck} b_{eff} d^2$

With Wisdom We Explore

Flange Beam

If applied moment $M < M_{bal}$, then compression reinforcement are not required. Area of tension reinforcement can be calculate as follows by taking moment at F_{cc2}.

$$M = F_{st}.z_2 - F_{cc1}.(z_2 - z_1)$$

$$= 0.87 f_{yk} A_s (d - 0.5h_f) - 0.454 f_{ck} b_w x[(d - 0.5h_f) - (d - 0.4x)]$$

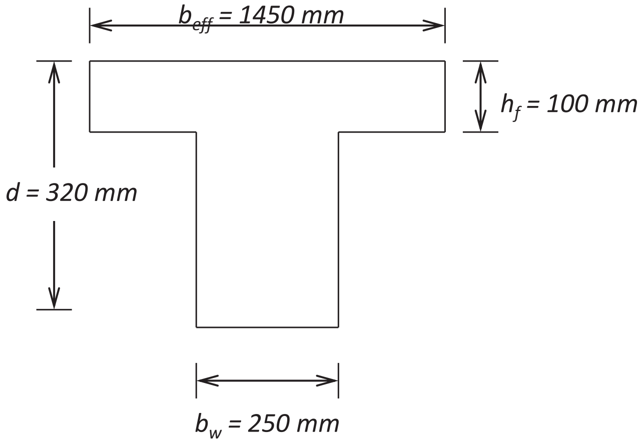
$$A_s = \frac{M + 0.454 f_{ck} b_w x[0.4x - 0.5h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

Using; $x = 0.45d$

$$A_s = \frac{M + 0.1 f_{ck} b_w d[0.36d - h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

Example 3.7

The T-beam with dimension as shown in figure below is subjected to design moment, $M = 670\ kNm$. If $f_{ck} = 30\ N/mm^2$ and $f_{yk} = 500\ N/mm^2$ have been used, determine the area and number of reinforcement required.



Solution of Example 3.7

Moment resistance of flange, M_f

$$M_f = (0.567 f_{ck} b_{eff} h_f)(d - h_f / 2)$$
$$= (0.567 \times 30 \times 1450 \times 100)(320 - 100/2) \times 10^{-6}$$
$$= 665.9 kNm < M = 670 kNm$$

Since $M > M_f$, Neutral axis lies in web

$$M_{bal} = \beta_f f_{ck} b_{eff} d^2$$

$$\beta_f = 0.167 \frac{250}{1450} + 0.567 \frac{100}{320}\left(1 - \frac{250}{1450}\right)\left(1 - \frac{100}{2(320)}\right) = 0.153$$

$$M_{bal} = 0.153(30)(1450)(320^2) \times 10^{-6} = 682 kNm$$

$$M_{bal} = 682 kNm > M = 670 kNm$$

Solution of Example 3.7

Compression reinforcement is not required

Area of tension reinforcement, A_s

$$A_s = \frac{M + 0.1 f_{ck} b_w d [0.36d - h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

$$= \frac{670 \times 10^6 + 0.1(30)(250)(320)[0.36(320) - 100]}{0.87(500)(320 - 50)}$$

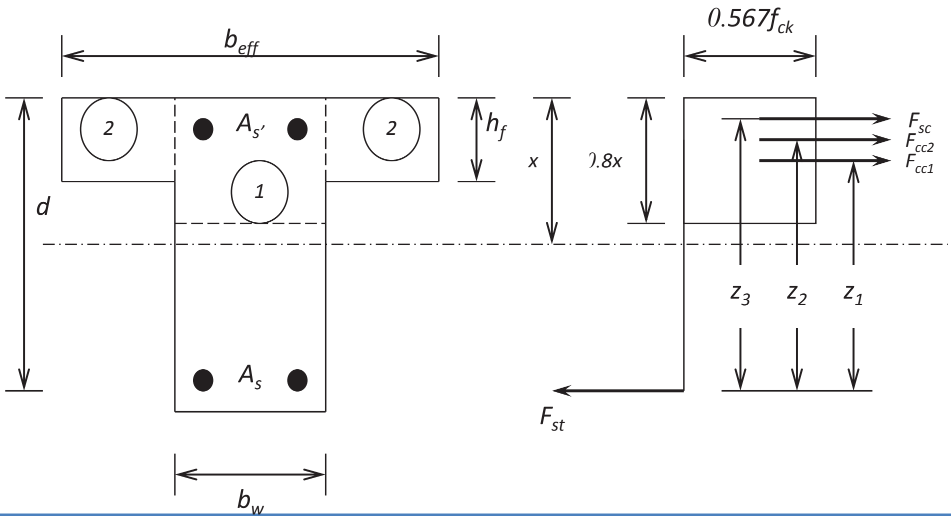
$$= 5736 mm^2$$

Provide 8H32 ($A_{sprov} = 6433\ mm^2$)

Flange Beam

Neutral axis lies in web $(M > M_{bal})$

- If the applied moment M is greater than M_{bal} the neutral axis lies in the web and the compression reinforcement should be provided. The stress block are shown in figure below.



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Flange Beam

From the stress block, internal forces;

$$F_{cc1} = (0.567 f_{ck})(b_w.0.8x) = 0.454 f_{ck} b_w x$$

$$F_{cc2} = (0.567 f_{ck})(b_{eff} - b_w) h_f$$

$$F_{sc} = 0.87 f_{yk} A_s'$$

$$F_{st} = 0.87 f_{yk} A_s$$

Lever arms, z

$$z_1 = d - 0.4x \qquad z_2 = d - 0.5h_f \qquad z_3 = d - d'$$

Moment resistance, M

$$M = F_{cc1}.z_1 + F_{cc2}.z_2 + F_{sc}.z_3$$

With Wisdom We Explore

Flange Beam

$$M = (0.454 f_{cu} b_w x)(d - 0.4x) + (0.567 f_{cu})(b_{eff} - b_w) h_f (d - 0.5h_f)$$
$$+ 0.87 f_{yk} A_s'(d - d')$$

When x = 0.45d, then

$$M = M_{bal} + 0.87 f_{yk} A_s'(d - d')$$

Area of compression reinforcement, A_s'

$$A_s' = \frac{M - M_{bal}}{0.87 f_{yk}(d - d')}$$

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Flange Beam

For equilibrium of forces

$$F_{st} = F_{cc1} + F_{cc2} + F_{sc}$$

$$0.87 f_{yk} A_s = 0.454 f_{ck} b_w (0.45d) + 0.567 f_{ck}(b_{eff} - b_w) + 0.87 f_{yk} A_s'$$

Area of tension reinforcement, A_s

$$A_s = \frac{0.2 f_{ck} b_w d + 0.567 f_{ck} h_f (b_{eff} - b_w)}{0.87 f_{yk}} + A_s'$$

Design Procedures for Rectangular Section

Supposed the design bending moment is M, beam section is b x d, concrete strength is f_{ck} and steel strength is f_{yk}, to determine the area of reinforcement, proceed as follows.

1. Calculate $K = \dfrac{M}{bd^2 f_{ck}}$

2. Calculate $K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$

where $\delta = \dfrac{\text{momen at section after redistribution}}{\text{momen at section before redistribution}} \leq 1.0$

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1. If $K \leq K_{bal}$, compression reinforcement is not required, and

   i. $z = d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right]$

   ii. $A_s = \dfrac{M}{0.87 f_{yk} z}$

2. If $K > K_{bal}$, compression reinforcement is required, and

   i. $z = d\left[0.5 + \sqrt{(0.25 - K_{bal}/1.134)}\right]$

   ii. $x = (d - z)/0.4$

   iii. Check d'/x

With Wisdom We Explore

$$A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')} \quad \text{if } d'/x \leq 0.38 \text{ or}$$

$$A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{f_{sc} (d - d')} \quad \text{if } d'/x > 0.38$$

where $f_{sc} = 700(1 - d'/x)$

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A_s' \left(\frac{f_{sc}}{0.87 f_{yk}} \right)$$

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Design Procedures for Flange Section

Supposed the design bending moment is M, beam section is b x d, concrete strength is f_{ck} and steel strength is f_{yk}, to determine the area of reinforcement, proceed as follows.

1. Calculate $M_f = 0.567 f_{ck} b h_f (d - 0.5 h_f)$

2. If $M \leq M_f$, neutral axis in the flange
   - i. $K = \dfrac{M}{b d^2 f_{ck}}$
   - ii. $z = d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right]$
   - iii. $A_s = \dfrac{M}{0.87 f_{yk} z}$

With Wisdom We Explore

Design Formula

1. If $M > M_f$, neutral axis in the flange

   i. Calculate $\beta_f = 0.167\dfrac{b_w}{b} + 0.567\dfrac{h_f}{d}\left(1-\dfrac{b_w}{b}\right)\left(1-\dfrac{h_f}{2d}\right)$

   ii. Calculate $M_{bal} = \beta_f f_{ck} b d^2$

   iii. Compare M and M_{bal}

2. If $M \leq M_{bal}$, compression reinforcement is not required.

   i. $A_s = \dfrac{M + 0.1 f_{ck} b_w d (0.36d - h_f)}{0.87 f_{yk}(d - 0.5h_f)}$

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Design Formula

1. If $M > M_{bal}$, compression reinforcement is required.

   i. $A_s' = \dfrac{(M - M_{bal})}{0.87 f_{yk}(d - d')}$

   ii. $A_s = \dfrac{0.167 f_{ck} b_w d + 0.567 f_{ck} h_f (b - b_w)}{0.87 f_{yk}} + A_s'$

With Wisdom We Explore