Consider a simply supported beam subjected to gradually increasing load. The load causes the beam to bend and exert a bending moment as shown in figure below.

- The top surface of the beam is seen to shorten under compression, and the bottom surface lengthens under tension.
- As the concrete cannot resist tension, steel reinforcement is introduced at the bottom surface to resist tension.
For continuous beam, the loads also cause the beam to bend downward between the support and upward bending over the support. This will produce tensile zone as shown in figure below. As the concrete cannot resist flexural tension, steel reinforcement would be introduced as detail in the figure.

- In the design of reinforced concrete beam the following assumptions are made (See EN 1991: Cl. 6.1 (2) P.)
  - Plane section through the beam before bending remain plane after bending.
  - The strain in bonded reinforcement, whether in tension or compression is the same as that in the surrounding concrete.
  - The tensile of the concrete is ignored.
  - The stresses in the concrete and reinforcement can be derived from the strain by using stress-strain curve for concrete and steel.
Basic Assumption in RC Design

- Plane sections remain plane, i.e.

The surface of any cross-section does not distort out-of-plane during deformation.

Distribution of Stresses and Strain

- Figure below shows the cross section of a RC beam subjected to bending and the resultant strain and stress distribution in the concrete.
- Top surface of cross section are subjected to compressive stresses while the bottom surface subjected to tensile stresses.
- The line that introduced in between the tensile and compression zones is known as the neutral axis of the member.
- Due to the tensile strength of concrete is very low, all the tensile stresses at the bottom fibre are taken by reinforcement.
For $f_{ck} < 50$ N/mm$^2$: $\eta = 1$ (defining the effective strength), $\varepsilon_c = 0.0035$,

$$\alpha_{cc} = 0.85, \lambda = 0.8, \gamma_c = 1.5,$$

$$f_{cd} = 1.0 \times 0.85 \times f_{ck} / 1.5 = 0.567 \times f_{ck}$$

Stress distribution in the concrete

The triangular stress distribution applies when the stress are very nearly proportional to the strain, which generally occurs at the loading levels encountered under working load conditions and is, therefore, used at the serviceability limit state.

The rectangular-parabolic stress block represents the distribution at failure when the compressive strain are within the plastic range, and it is associated with the design for ultimate limit state.

The equivalent rectangular stress block is a simplified alternative to the rectangular-parabola distribution.
The distribution of strains across the beam cross section is linear. That is, the normal strain at any points in a beam section is proportional to its distance from the neutral axis.

- The steel strain in tension $\varepsilon_{st}$ can be determined from the strain diagram as follows:

$$\frac{\varepsilon_{st}}{(d-x)} \cdot \frac{x}{\varepsilon_{cc}} \Rightarrow \varepsilon_{st} = \varepsilon_{cc} \left( \frac{d-x}{x} \right)$$

Therefore:

$$x = \frac{d}{1 + \left( \frac{\varepsilon_{st}}{\varepsilon_{cc}} \right)}$$

- Since $\varepsilon_{cc} = 0.0035$ for class $\leq$ C50/60 and
- For steel with $f_{yk} = 500$ N/mm$^2$ and the yield strain is $\varepsilon_{st} = 0.00217$.
- By substituting $\varepsilon_{cc}$ and $\varepsilon_{st}$, $x = 0.617d$

Hence, to ensure yielding of the tension steel at limit state the depth of neutral axis, $x$ should be less than or equal to $0.617d$. 
As applied moment on the beam section increased beyond the linear elastic stage, the concrete strains and stresses enter the nonlinear stage.

- The behavior of the beam in the nonlinear stage depends on the amount of reinforcement provided.
- The reinforcing steel can sustain very high tensile strain however, the concrete can accommodate compressive strain much lower compare to it.
- So, the final collapse of a normal beam at ultimate limit state is cause by the crushing of concrete in compression, regardless of whether the tension steel has yield or not.

Depending on the amount of reinforcing steel provided, flexural failure may occur in three ways:

- **Balanced**: Concrete crushed and steel yields simultaneously at the ultimate limit state. The compressive strain of concrete reaches the ultimate strains $\varepsilon_{cu}$ and the tensile strain of steel reaches the yield strain $\varepsilon_y$ simultaneously. The depth of neutral axis, $x = 0.617d$.

- **Under-reinforced**: Steel reinforcement yields before concrete crushes. The area of tension steel provided is less than balance section. The depth of neutral axis, $x < 0.617d$. The failure is gradual, giving ample prior warning of the impending collapse. This mode if failure is preferred in design practice.

- **Over-reinforced**: Concrete fails in compression before steel yields. The area of steel provided is more than area provided in balance section. The depth of neutral axis, $x > 0.617d$. The failure is sudden (without any sign of warning) and brittle. Over-reinforced are not permitted.
For a singly reinforced beam EC2 limits the depth to the neutral axis, \( x \) to 0.45\( d \) \((x \leq 0.45d)\) for concrete class \( \leq \text{C50/60} \) to ensure that the design is for the under-reinforced case where failure is gradual, as noted above. For further understanding, see the graph shown below.

---

- **Section 6.1 EN 1992-1-1**, deal with the analysis and design of section for the ultimate limit state design consideration of structural elements subjected to bending.
- The two common types of reinforced concrete beam section are:
  - Rectangular section: Singly and doubly reinforced
  - Flanged section: Singly and doubly reinforced

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![Graph showing load and deflection with points indicating concrete crack without steel yield and steel yield](image-url)

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![Images of different types of beams](image-url)

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Beam cross section, strains and stresses distribution at ULS of singly reinforced rectangular beam

Notation:

- $h$ = Overall depth
- $b$ = Width of section
- $A_s$ = Area of tension reinforcement
- $f_{ck}$ = Characteristic strength of concrete
- $f_{yk}$ = Characteristic strength of reinforcement
- $d$ = Effective depth
- $s$ = Depth of stress block
- $x$ = Neutral axis depth
- $z = d - 0.5s$

Tension force of steel, $F_{st}$

$$F_{st} = \text{Stress} \times \text{Area} = 0.87 f_{yk} A_s$$

Compression force of concrete, $F_{cc}$

$$F_{cc} = \text{Stress} \times \text{Area} = 0.567 f_{ck} (b \times 0.8x) = 0.454 f_{ck} bx$$

For equilibrium, total force in the section should be zero.

$$F_{cc} = F_{st}$$

$$0.454 f_{ck} bx = 0.87 f_{yk} A_s$$

$$x = \frac{0.87 f_{yk} A_s}{0.454 f_{ck} b}$$
Moment resistance with respect to the steel
\[ M = F_{cc} \times z \]
\[ M = (0.454 f_{ck} b x)(d - 0.4x) = \left( \frac{0.454 x}{d} \right) \left( \frac{d - 0.4x}{d} \right) (f_{ck} b d^2) \]

Let's; \[ \left( \frac{0.454 x}{d} \right) \left( 1 - \frac{0.4x}{d} \right) = K \]
Therefore; \[ M = K \cdot f_{ck} \cdot b \cdot d^2 \]

Moment resistance with respect to the concrete
\[ M = F_{st} \times z \]
\[ M = (0.87 f_{yk} A_s)(d - 0.4x) \]

Area of tension reinforcement, \[ A_s = \frac{M}{0.87 f_{yk} \cdot (d - 0.4x)} \]

To ensure that the section designed is under-reinforced it is necessary to place a limit on the maximum depth of the neutral axis \((x)\). EC2 suggests:
\[ x \leq 0.45d \]

Then ultimate moment resistance of singly reinforced section or \( M_{bal} \) can be obtained by;
\[ M_{bal} = (0.454 f_{ck} b x)(d - 0.4x) \]
\[ M_{bal} = [0.454 \cdot f_{ck} \cdot b(0.45d)] \cdot [d - 0.4(0.45d)] \]
\[ M_{bal} = (0.2043 \cdot f_{ck} \cdot b \cdot d) \cdot (0.82d) \]
\[ M_{bal} = 0.167 \cdot f_{ck} \cdot b \cdot d^2 \cdot K_{bal} \cdot f_{ck} \cdot b \cdot d^2 \]
Therefore;

\[ M = K f_{ck} b d^2 \]
\[ M_{bal} = K_{bal} f_{ck} b d^2 \]

where; \( K_{bal} = 0.167 \)

If;

\[ M \leq M_{bal} \text{ or } K \leq K_{bal} \] : Singly reinforced rectangular beam
(Tension reinforcement only)

\[ M > M_{bal} \text{ or } K > K_{bal} \] : Doubly reinforced rectangular beam
(Section requires compression reinforcement)

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**Example 3.1**

The cross section of rectangular beam is shown in figure below. Using stress block diagram and the data given, determine the area and the number of reinforcement required.

**Data:**
- Design moment, \( M_{ED} \) = 200 kN.m
- \( f_{ck} \) = 25 N/mm\(^2\)
- \( f_{yk} \) = 500 N/mm\(^2\)
- \( b = 250 \text{ mm} \)
- \( d = 450 \text{ mm} \)
Calculate the ultimate moment resistance of section, $M_{bal}$

$$M_{bal} = 0.167 \cdot f_{ck} \cdot b \cdot d^2$$

$$= 0.167 \cdot 25 \cdot 250 \cdot (450^2)$$

$$= 211.36 \text{kNm} > M = 200 \text{kNm}$$

Singly reinforced section
Neutral axis depth, $x$

$$M = (0.454 \cdot f_{ck} \cdot b \cdot x)(d - 0.4x)$$

$$200 \times 10^6 = 0.454 \cdot 25 \cdot 250 \cdot x \cdot (450 - 0.4x)$$

$$x^2 - 1125x + 176211.45 = 0$$

$x = 188 \text{ mm} @ 937 \text{ mm}$

Use $x = 188 \text{ mm}$

Checking: \[ \frac{x}{d} = \frac{188}{450} = 0.42 < 0.45 \]

Lever arm, $z = (d - 0.4x)$

$$= (450 - 0.4(188)) = 374.8 \text{ mm}$$

Area of reinforcement, $A_s$

$$A_s = \frac{M}{0.87 \cdot f_{yk} \cdot z} = \frac{200 \times 10^6}{0.87 \cdot 500 \cdot (374.8)} = 1227 \text{ mm}^2$$

Provide 4H20 ($A_{sprov} = 1257 \text{ mm}^2$)
Figure below shows the cross section of a singly reinforced beam. Determine the resistance moment for that cross section with the assistance of a stress block diagram. Given $f_{ck} = 25 \text{ N/mm}^2$ and $f_{yk} = 500 \text{ N/mm}^2$.

A stress block diagram is drawn with the important values and notations.

For equilibrium;

$$F_{cc} = F_{st}$$

$$0.454f_{ck} bx = 0.87f_{yk} A_s$$

$$x = \frac{0.87 f_{yk} A_s}{0.454 f_{ck} b}$$
Solution of Example 3.2

Checking;

\[
x = \frac{0.87(500)(982)}{0.454(25)(250)} = 151 \text{ mm}
\]

\[
\frac{x}{d} = \frac{151}{450} = 0.34 < 0.45
\]

Moment resistance of section;

\[
M = F_{cc} \times z \quad @ \quad M = F_{st} \times z
\]

\[
M = (0.454 f_{ck} b x)(d - 0.4x)
\]

\[
M = (0.454 \times 25 \times 250 \times 151)(450 - 0.4(151))
\]

\[
M = 167 \text{ kNm}
\]

With Wisdom We Explore

Doubly Reinforced Rectangular Beam

- When the load applied increases gradually and it will reach a state that the compressive strength of concrete is not adequate to take additional compressive stress.
- Compression reinforcement is required to take the additional compressive stress.
- This section is named as doubly reinforced section.
Strain and stress block diagrams of doubly reinforced beam.

Internal force;

\[ F_{cc} = 0.454 f_{ck} b x \]
\[ F_{st} = 0.87 f_{yk} A_s \]
and
\[ F_{sc} = 0.87 f_{yk} A'_s \]

Lever arms;

\[ z = d - 0.4x \]
\[ z_1 = d - d' \]

For equilibrium of internal force;

\[ F_{st} = F_{cc} + F_{sc} \]
\[ 0.87 f_{yk} A_s = 0.454 f_{ck} b x + 0.87 f_{yk} A'_s \]
Taking moment about the centroid of the tension steel,

\[ M = F_{cc} \cdot z + F_{sc} \cdot z_1 \]

\[ M = (0.454 f_{ck} b x)(d - 0.4x) + (0.87 f_{yk} A_s').(d - d') \]

For design purpose, \( x = 0.45d \)

\[ M = (0.454 f_{ck} b x).[d - 0.4(0.45d)] + (0.87 f_{yk} A_s').(d - d') \]

\[ = 0.167 f_{ck} b d^2 + (0.87 f_{yk} A_s').(d - d') \]

\[ = M_{bal} + (0.87 f_{yk} A_s').(d - d') \]

The area of compression reinforcement, \( A_s' \)

\[ A_s' = \frac{(M - M_{bal})}{0.87 f_{yk} (d - d')} \quad \text{or} \quad A_s' = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')} \]

The area of tension reinforcement, \( A_s \)

Multiplied equilibrium internal force equation by \( z \),

Limiting \( x = 0.45d \) and \( z = d - 0.4(0.45d) = 0.82d \)

\[ 0.87 f_{yk} A_s z = 0.454 f_{ck} b x z + 0.87 f_{yk} A_s' z \]

\[ 0.87 f_{yk} A_s z = 0.454 f_{ck} b (0.45d)(0.82d) + 0.87 f_{yk} A_s' z \]

\[ 0.87 f_{yk} A_s z = 0.167 f_{ck} b d^2 + 0.87 f_{yk} A_s' z \]

\[ A_s = \frac{0.167 f_{ck} b d^2}{0.87 f_{yk} z} + A_s' \quad \text{or} \quad A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A_s' \]
Stress in compression reinforcement.

- The derivation of design formula for doubly reinforced section assumed that the compression reinforcement reaches the design strength of $0.87f_{yk}$ at ultimate limit state.
- From the strain diagram as shown in figure below.

For the design strength $0.87f_{yk}$ to be reached, $\varepsilon_{sc} = \frac{0.87f_{yk}}{E_s}$

$$\varepsilon_{sc} = \frac{0.87f_{yk}}{E_s} = \frac{0.87(500)}{200 \times 10^3} = 0.002175$$

$$\frac{d'}{x} = 1 - \left(\frac{0.002175}{0.0035}\right) = 0.38$$

Therefore, if $d'/x < 0.38$ the compression reinforcement can be assumed reach the design strength of $0.87f_{yk}$. If $d'/x > 0.38$, a reduced stress should be used.

$$f_{sc} = E_s \cdot \varepsilon_{sc}$$

$$f_{sc} = 200 \times 10^3 (0.0035)(1 - d'/x) = 700(1 - d'/x)$$
The cross section of rectangular beam is shown in figure below. Using the data given, determine the area and the number of reinforcement required.

**Data:**
- Design moment, $M_{Ed} = 450\, \text{kN.m}$
- $f_{ck} = 25\, \text{N/mm}^2$
- $f_{yk} = 500\, \text{N/mm}^2$
- $d' = 50\, \text{mm}$

**Solution of Example 3.3**

Ultimate moment resistant of section, $M_{bal}$

$$M_{bal} = 0.167 \cdot f_{ck} \cdot b \cdot d^2$$

$$= 0.167(25)(250)(500^2)(10^{-6})$$

$$= 260.94\, \text{kNm} < M = 450\, \text{kNm}$$

Compression reinforcement is required

Area of compression reinforcement, $A_{s'}$

$$A_{s'} = (M - M_{bal}) / 0.87 \cdot f_{yk} (d - d')$$

$$= (450 - 260.94) \times 10^6 / 0.87(500)(500 - 50)$$

$$= 966\, \text{mm}^2$$
Checking $d'/x$ ratio

$$x = 0.45d = 0.45(500) = 225 \text{mm}$$

$$d'/x = 50 / 225 = 0.22 < 0.38$$

Compression steel achieved it design strength at $0.87f_{yk}$

Area of tension steel, $A_s$

$$A_s = \left( \frac{M_{bal}}{0.87f_{yk}x} \right) + A_{s'} = \frac{260.94 \times 10^6}{0.87 \times 500 \times (0.82 \times 500)} + 966$$

$$= 2429 \text{mm}^2$$

Provide $2H25$ ($A_{s\text{prov}} = 982 \text{ mm}^2$) – Compression reinforcement

$5H25$ ($A_{s\text{prov}} = 2454 \text{ mm}^2$) – Tension reinforcement

Calculate moment resistance of the doubly reinforced section shown in figure below. Given $f_{ck} = 30 \text{ N/mm}^2$ and $f_{yk} = 500 \text{ N/m}^2$. 

With Wisdom We Explore
A stress block diagram is drawn with the important values and notations.

Reinforcement used 3H20, \( A_s' = 943 \text{ mm}^2 \) & 5H25, \( A_s = 2455 \text{ mm}^2 \)

Neutral axis depth, \( x \)

\[
x = \frac{0.87 f_{yk} (A_s - A_s')}{{0.454 f_{ck} b}} = \frac{0.87(500)(2455 - 943)}{0.454(30)(250)}
\]

\( x = 193 \text{mm} \)

Checking the stress of steel

\( x / d = 193 / 500 = 0.39 < 0.45 \)

\( d'/x = 50 / 193 = 0.26 < 0.38 \)

Steel achieved its design strength 0.87\( f_y \) as assumed.
Moment resistance of section, $M$

$$M = F_{sc} \cdot z_1 + F_{cc} \cdot z$$

$$= 0.87 f_{yk} A_s \left( d - d' \right) + 0.454 f_{ck} b x (d - 0.4x)$$

$$= 0.87(500)(943)(500 - 50) + 0.454(30)(250)(193)(500 - 0.4(193)) \times 10^{-6}$$

$$= 462 kNm$$

---

Flanged beams occur when beams are cast integrally with and support a continuous floor slab.

Part of the slab adjacent to the beam is counted as acting in compression to form T- and L-beams as shown in figure below.

Where:
- $b_{eff}$ = effective flange width
- $b_w$ = breadth of the web of the beam.
- $h_f$ = thickness of the flange.
The effective width of flange, $b_{\text{eff}}$ is given in Sec. 5.3.2.1 of EC2.

$b_{\text{eff}}$ should be based on the distance $l_0$ between points of zero moment as shown in figure below.

\[ l_0 = 0.85 \, h \quad l_0 = 0.15 (l_1 + l_2) \quad l_0 = 0.7 \, l_2 \quad l_0 = 0.15 \, l_2 + l_3 \]

---

The effective flange width, $b_{\text{eff}}$ for T-beam or L-beam may be derived as:

\[ b_{\text{eff}} = \Sigma b_{\text{eff,i}} + b_w \leq b \]

Where

\[ b_{\text{eff,i}} = 0.2b_i + 0.1l_0 \leq 0.2l_0 \quad \text{and} \quad b_{\text{eff,i}} \leq b_i \]
Based on figure below, determine the effective flange width, $b_{eff}$ of beam B/1-3.

**Solution of Example 3.5**

$I_o$ (distance between points of zero moment)

\[ I_o = 0.85 \times 3000 = 2550 \, \text{mm} \]

\[ I_o = 0.15 \times (3000 + 4500) = 1125 \, \text{mm} \]

\[ I_o = 0.85 \times 4500 = 3825 \, \text{mm} \]

Effective flange width, $b_{eff}$

\[ b_{eff} = \sum b_{eff,i} + b_w \leq b \]
Solution of Example 3.5

Span 1-2

\[ b_{\text{eff}1} = 0.2(1250) + 0.1(2550) = 505 \text{ mm} < 0.2I_o = 510 \text{ mm} < b_1 = 1250 \text{ mm} \]

\[ b_{\text{eff}2} = 0.2(2000) + 0.1(2550) = 655 \text{ mm} > 0.2I_o = 510 \text{ mm} < b_2 = 2000 \text{ mm} \]

\[ b_{\text{eff}} = (505 + 510) + 200 = 1215 \text{ mm} < 3250 \text{ mm} \]

Span 2-3

\[ b_{\text{eff}1} = 0.2(1250) + 0.1(3825) = 632.5 \text{ mm} < 0.2I_o = 765 \text{ mm} < b_1 = 1250 \text{ mm} \]

\[ b_{\text{eff}2} = 0.2(2000) + 0.1(3825) = 782.5 \text{ mm} > 0.2I_o = 765 \text{ mm} < b_2 = 2000 \text{ mm} \]

\[ b_{\text{eff}} = (632.5 + 765) + 200 = 1597.5 \text{ mm} < 3250 \text{ mm} \]
The design procedure of flange beam depends on where the neutral axis lies. The neutral axis may lie in the flange or in the web.

In other word, there are three cases that should be considered.

- Neutral axis lies in flange \((M < M_f)\)
- Neutral axis lies in web \((M > M_f \text{ but } M < M_{bal})\)
- Neutral axis lies in web \((M > M_{bal})\)
Neutral axis lies in flange \((M < M_f)\)

- This condition occurs when the depth of stress block \((0.8 \times)\) less than the thickness of flange, \(h_f\) as shown in figure below.

\[
M = F_{cc} \times z
\]

\[
M = \left(0.567 f_{cu} b_{eff} 0.8x\right)(d - 0.4x)
\]

For this case, maximum depth of stress block, \(0.8x\) are equal to \(h_f\)

\[
M = M_f = \left(0.567 f_{ck} bh_f\right)(d - h_f / 2)
\]

Where, \(M_f\) = Ultimate moment resistance of flange.

Therefore, if \(M \leq M_f\) the neutral axis lies in flange and the design can be treated as rectangular singly reinforced beam.

\[
A_s = \frac{M}{0.87 f_{yk} z} \quad \text{or} \quad A_s = \frac{M}{0.87 f_{yk} (d - 0.4x)}
\]
The T-beam with dimension as shown in figure below is subjected to design moment, $M = 250 \text{kNm}$. If $f_{ck} = 30 \text{N/mm}^2$ and $f_{yk} = 500 \text{N/mm}^2$ have been used, determine the area and number of reinforcement required.

**Solution of Example 3.6**

Moment resistance of flange, $M_f$

$$M_f = (0.567 f_{ck} b_{eff} h_f)(d - h_f / 2)$$

$$= (0.567 \times 30 \times 1450 \times 100)(320 - 100 / 2) \times 10^{-6}$$

$$= 665.9 \text{kNm} > M = 250 \text{kNm}$$

Since $M < M_f$, Neutral axis lies in flange

Compression reinforcement is not required

$$M = (0.454 f_{ck} b x)(d - 0.4x)$$

$$250 \times 10^6 = 0.454(30)(1450)(x)(320 - 0.4x)$$

$$x^2 - 800x + 31647.2 = 0$$

$$x = 758.3 \text{mm} @ 41.74 \text{mm} \text{ Use } x = 41.74 \text{mm}$$
Checking

\[ \frac{x}{d} = \frac{41.74}{320} = 0.13 < 0.45 \]

Lever arm, \( z \)

\[ z = d - 0.4x = 320 - 0.4(41.74) = 303.3\text{mm} \]

Area of tension reinforcement, \( A_s \)

\[ A_s = \frac{M}{0.87f_{yk}z} = \frac{250 \times 10^6}{0.87(500)(303.3)} \]

\[ = 1895\text{mm}^2 \]

*Provide 4H25 (\( A_{sprov} = 1964\text{mm}^2 \))*

---

**Neutral axis lies in web \( (M_f < M < M_{bal}) \)**

- If the applied moment \( M \) is greater than \( M_f \) the neutral axis lies in the web as shown in figure below.
From the stress block, internal forces:

\[ F_{cc1} = (0.567 f_{ck}) (b_w \cdot 0.8x) = 0.454 f_{ck} b_w x \]
\[ F_{cc2} = (0.567 f_{ck}) (b_{eff} - b_w) h_f \]
\[ F_{st} = 0.87 f_{yk} A_s \]

Lever arms, \( z \)

\[ z_1 = d - 0.4x \quad z_2 = d - 0.5h_f \]

Moment resistance, \( M \)

\[ M = F_{cc1} z_1 + F_{cc2} z_2 \]
\[ = (0.454 f_{ck} b_w x) (d - 0.4x) + (0.567 f_{ck}) (b_{eff} - b_w) h_f (d - 0.5h_f) \]

Ultimate moment resistance of section, \( M_{bal} \) (When \( x = 0.45d \))

\[ M_{bal} = (0.454 f_{ck} b_w 0.45d) (d - 0.4(0.45d)) \]
\[ + (0.567 f_{ck}) (b_{eff} - b_w) h_f (d - 0.5h_f) \]

Divide both side by \( f_{ck} b_{eff} d^2 \), then;

\[ \frac{M_{bal}}{f_{ck} b_{eff} d^2} = 0.167 \frac{b_w}{b_{eff}} + 0.567 \frac{h_f}{d} \left( 1 - \frac{b_w}{b_{eff}} \right) \left( 1 - \frac{h_f}{2d} \right) \]

\[ \frac{M_{bal}}{f_{ck} b_{eff} d^2} = \beta_f \]

Therefore;

\[ M_{bal} = \beta_f f_{ck} b_{eff} d^2 \]
If applied moment $M < M_{bal}$, then compression reinforcement are not required. Area of tension reinforcement can be calculate as follows by taking moment at $F_{cc2}$:

$$M = F_{st}z_2 - F_{cc1}(z_2 - z_1)$$

$$= 0.87 f_{yk} A_s (d - 0.5h_f) - 0.454 f_{ck} b_w x [(d - 0.5h_f) - (d - 0.4x)]$$

$$A_s = \frac{M + 0.454 f_{ck} b_w x [0.4x - 0.5h_f]}{0.87 f_{yk} (d - 0.5h_f)}$$

Using: $x = 0.45d$

$$A_s = \frac{M + 0.1 f_{ck} b_w d [0.36d - h_f]}{0.87 f_{yk} (d - 0.5h_f)}$$

Example 3.7

The T-beam with dimension as shown in figure below is subjected to design moment, $M = 670 \text{kNm}$. If $f_{ck} = 30 \text{ N/mm}^2$ and $f_{yk} = 500 \text{ N/mm}^2$ have been used, determine the area and number of reinforcement required.

![T-beam diagram](https://example.com/tbeam.png)
Moment resistance of flange, $M_f$

$$M_f = \left(0.567 f_{ck} b_{eff} h_f \right) \left(d - h_f / 2\right)$$

$$= (0.567 \times 30 \times 1450 \times 100)(320 - 100 / 2) \times 10^{-6}$$

$$= 665.9 kNm < M = 670 kNm$$

Since $M > M_f$, neutral axis lies in web

$$M_{bal} = \beta_f f_{ck} b_{eff} d^2$$

$$\beta_f = 0.167 \frac{250}{1450} + 0.567 \frac{100}{320} \left(1 - \frac{250}{1450}\right) \left(1 - \frac{100}{2(320)}\right) = 0.153$$

$$M_{bal} = 0.153(30)(1450)(320^2) \times 10^{-6} = 682 kNm$$

$$M_{bal} = 682 kNm > M = 670 kNm$$

Compression reinforcement is not required

Area of tension reinforcement, $A_s$

$$A_s = \frac{M + 0.1 f_{ck} b_w d[0.36d - h_f]}{0.87 f_{yk} (d - 0.5h_f)}$$

$$= \frac{670 \times 10^6 + 0.1(30)(250)(320)[0.36(320) - 100]}{0.87(500)(320 - 50)}$$

$$= 5736 mm^2$$

Provide 8H32 ($A_{sprov} = 6433 mm^2$)
Neutral axis lies in web \((M > M_{bal})\)

- If the applied moment \(M\) is greater than \(M_{bal}\) the neutral axis lies in the web and the compression reinforcement should be provided. The stress block are shown in figure below.

From the stress block, internal forces;

\[
F_{cc1} = (0.567 f_{ck})(b_w \cdot 0.8x) = 0.454 f_{ck} b_w x
\]

\[
F_{cc2} = (0.567 f_{ck})(b_{eff} - b_w) h_f
\]

\[
F_{sc} = 0.87 f_{yk} A_s'
\]

\[
F_{st} = 0.87 f_{yk} A_s
\]

Lever arms, \(z\)

\[
z_1 = d - 0.4x \quad z_2 = d - 0.5h_f \quad z_3 = d - d'
\]

Moment resistance, \(M\)

\[
M = F_{cc1} \cdot z_1 + F_{cc2} \cdot z_2 + F_{sc} \cdot z_3
\]
When \( x = 0.45d \), then

\[
M = (0.454 f_{cu} b_w x)(d - 0.4x) + (0.567 f_{cu})(b_{eff} - b_w)h_f (d - 0.5h_f) + 0.87 f_{yk} A_s'(d - d')
\]

When \( x = 0.45d \), then

\[
M = M_{bal} + 0.87 f_{yk} A_s'(d - d')
\]

Area of compression reinforcement, \( A_s' \)

\[
A_s' = \frac{M - M_{bal}}{0.87 f_{yk} (d - d')}
\]

For equilibrium of forces

\[
F_{st} = F_{ec1} + F_{cc2} + F_{sc}
\]

\[
0.87 f_{yk} A_s = 0.454 f_{ck} b_w (0.45d) + 0.567 f_{ck} (b_{eff} - b_w) + 0.87 f_{yk} A_s'
\]

Area of tension reinforcement, \( A_s \)

\[
A_s = \frac{0.2 f_{ck} b_w d + 0.567 f_{ck} h_f (b_{eff} - b_w)}{0.87 f_{yk}} + A_s'
\]
Design Procedures for Rectangular Section

Supposed the design bending moment is $M$, beam section is $b \times d$, concrete strength is $f_{ck}$ and steel strength is $f_{yk}$, to determine the area of reinforcement, proceed as follows.

1. Calculate
   \[ K = \frac{M}{bd^2 f_{ck}} \]

2. Calculate
   \[ K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2 \]
   where \[ \delta = \frac{\text{momen at section after redistribution}}{\text{momen at section before redistribution}} \leq 1.0 \]

   1. If $K \leq K_{bal}$, compression reinforcement is not required, and
      i. $z = d\left[0.5 + \sqrt{(0.25 - K/1.134)}\right]$
      ii. $A_s = \frac{M}{0.87f_{yk}z}$

   2. If $K > K_{bal}$, compression reinforcement is required, and
      i. $z = d\left[0.5 + \sqrt{(0.25 - K_{bal}/1.134)}\right]$
      ii. $x = (d - z)/0.4$
      iii. Check $d'/x$
Supposed the design bending moment is $M$, beam section is $b \times d$, concrete strength is $f_{ck}$ and steel strength is $f_{yk}$, to determine the area of reinforcement, proceed as follows.

1. Calculate $M_f = 0.567f_{ck} \cdot bh_f (d - 0.5h_f)$

2. If $M \leq M_f$, neutral axis in the flange
   i. $K = \frac{M}{bd^2 f_{ck}}$
   ii. $z = d \left[0.5 + \sqrt{(0.25 - \frac{K}{1.134})}\right]$
   iii. $A_s = \frac{M}{0.87f_{yk}z}$
1. If $M > M_f$, neutral axis in the flange
   i. Calculate $\beta_f = \frac{0.167 b_w}{b} + 0.567 \frac{h_t}{d} \left(1 - \frac{b_w}{b} \right) \left(1 - \frac{h_t}{2d} \right)$
   ii. Calculate $M_{bal} = \beta_f f_{ck} b d^2$
   iii. Compare $M$ and $M_{bal}$

2. If $M \leq M_{bal}$, compression reinforcement is not required.
   i. $A_s = \frac{M + 0.1 f_{ck} b_w d (0.36 d - h_t)}{0.87 f_{yk} (d - 0.5 h_t)}$

1. If $M > M_{bal}$, compression reinforcement is required.
   i. $A_s' = \frac{(M - M_{bal})}{0.87 f_{yk} (d - d')}$
   ii. $A_s = \frac{0.167 f_{ck} b_w d + 0.567 f_{ck} h_t (b - b_w)}{0.87 f_{yk}} + A_s'$