CHAPTER 2

STRUCTURAL ANALYSIS

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Introduction

- The primary purpose of structural analysis is to establish the distribution of internal forces and moments over the whole part of a structure and to identify the critical design conditions at all sections.
- The type of analysis should be appropriate to the problem being considered. The following may be used: linear elastic analysis, linear elastic analysis with limited redistribution, and plastic analysis.
- Linear elastic analysis may be carried out assuming cross sections are uncracked (i.e. concrete section properties), using linear stress-strain relationships, and assuming means values of elastic modulus.

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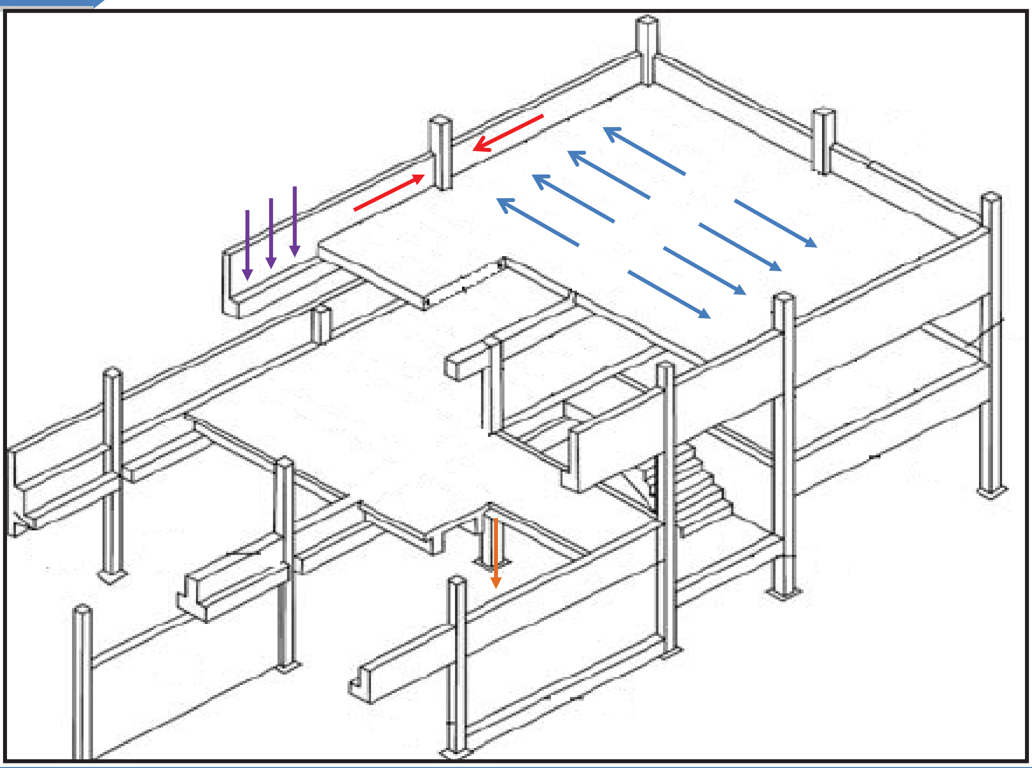
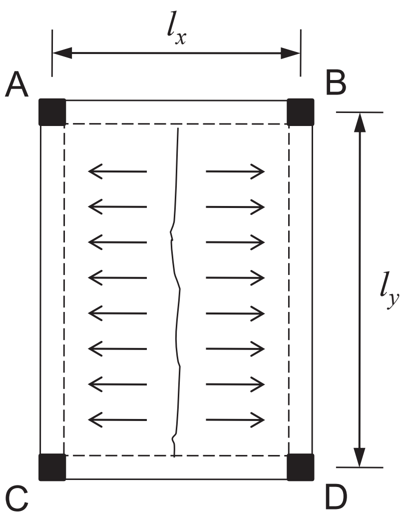
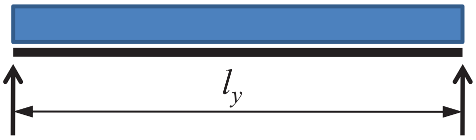
- Actions that applied on a beam may consist of beams self-weight, permanent and variable actions from slabs, actions from secondary beams and other structural or non-structural members supported by the beam.
- The distribution of slab actions on beams depends on the slab dimension, supporting system and boundary condition.
- Beam supporting slabs designed as spanning one-way can be considered to be uniformly load as shown in figure below.



Analysis of Actions

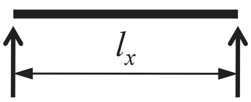


$w = 0.5.n.l_x$ kN/m



Beam AC and BD

$w = 0$ kN/m



Beam AB and CD

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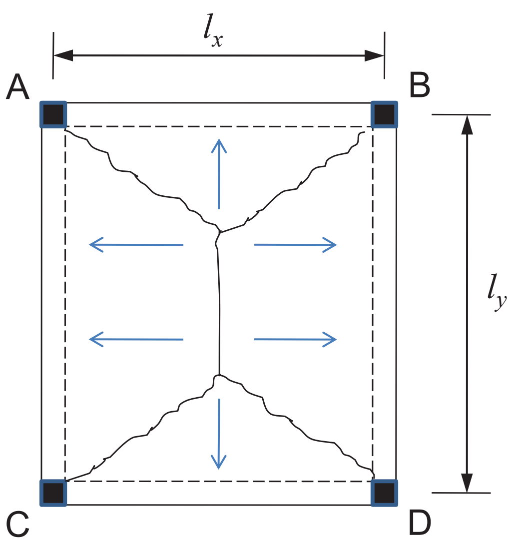
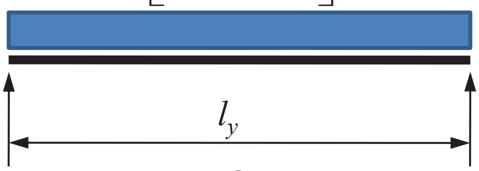
Analysis of Actions

- For beam supporting a two-way slab panel freely supported along four edge.

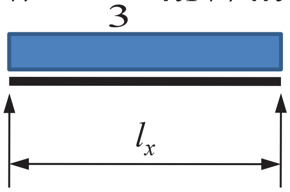


$$w = \frac{nl_x}{6}\left[3 - \left(\frac{l_x}{l_y}\right)^2\right] kN/m$$



Beam AC and BD

$$w = \frac{nl_x}{3} kN/m$$



Beam AB and CD

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- There are alternatives methods which consider various support conditions and slab continuity. The methods are, (i). Slab shear coefficient from **Table 3.15 BS 8110**, (ii). Yield line analysis and (iii). Table 63 Reinforced Concrete Designer's Handbook by Reynold.

Table 3.15 — Shear force coefficient for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners

Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	—	—	—	—	—	—	—	—	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—

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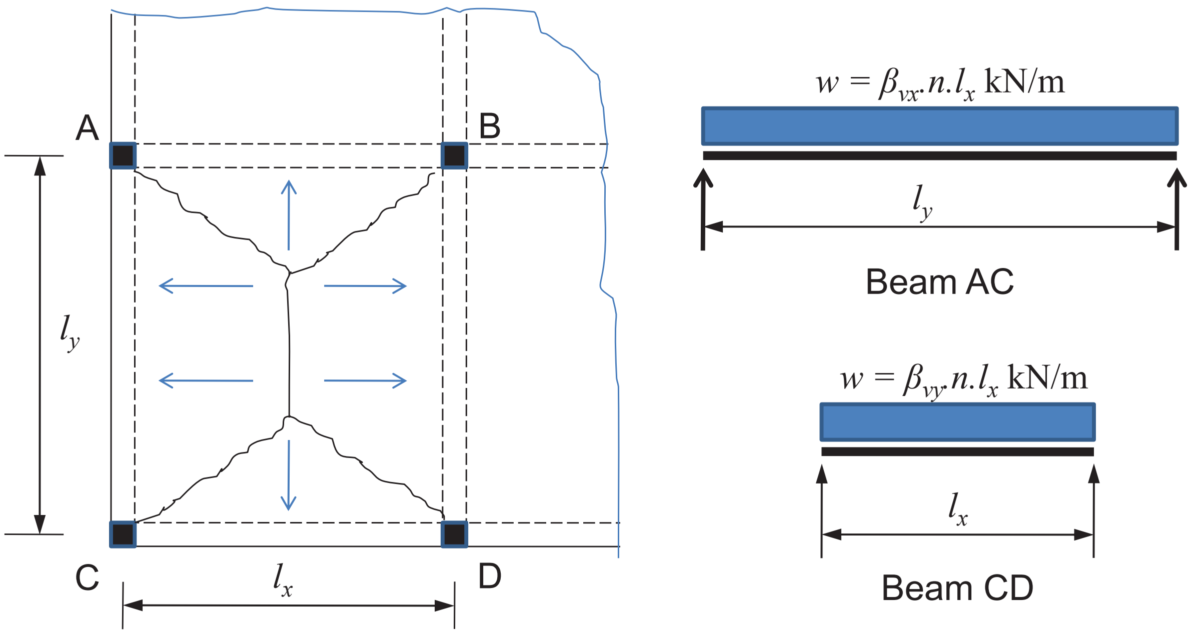

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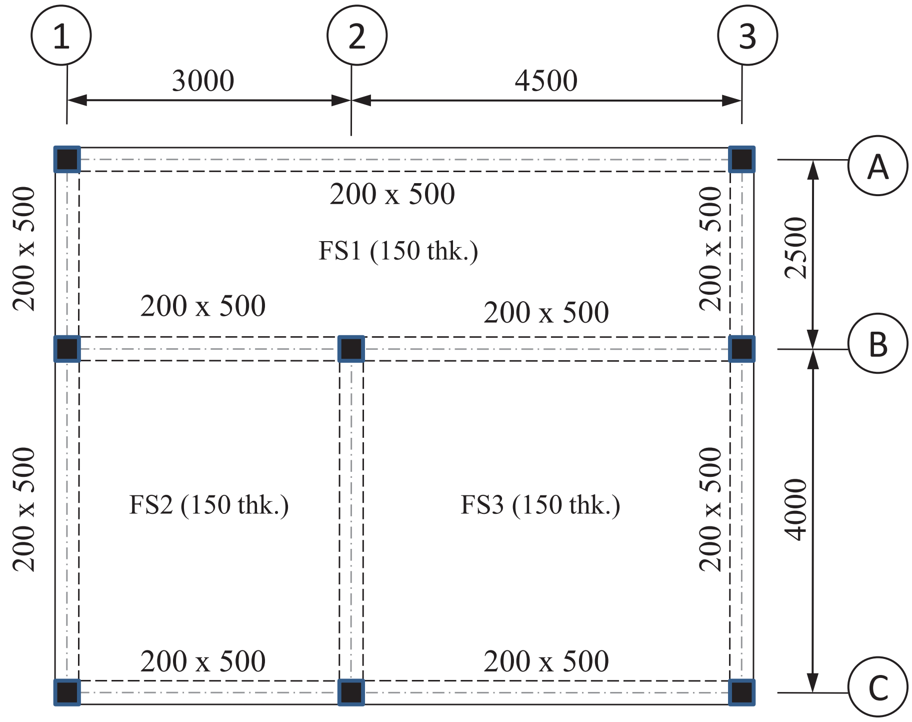
Example 2.1

Determine the characteristic permanent and variable action act on beam B/1-3.



Weight of concrete = 25 kN/m^3
Finishes, ceiling and services = 2.0 kN/m^2
Variable action (All slab) = 3.0 kN/m^2

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Solution of Example 2.1

Action on slab

Selfweight	= 0.15 x 25	= 3.75 kN/m^2
Finishes, ceiling and services		= 2.0 kN/m^2
Chac. Permanent action, G_k		**= 5.75 kN/m^2**
Chac. Variable action, Q_k		**= 3.0 kN/m^2**

Distribution of actions from slabs are as follows;

FS1 : l_y/l_x = 7.5 / 2.5 = 3 > 2.0, One-way slab

FS2 : l_y/l_x = 4.0 / 3.0 = 1.33 < 2.0, Two-way slab

FS3 : l_y/l_x = 4.5 / 4.0 = 1.13 < 2.0, Two-way slab

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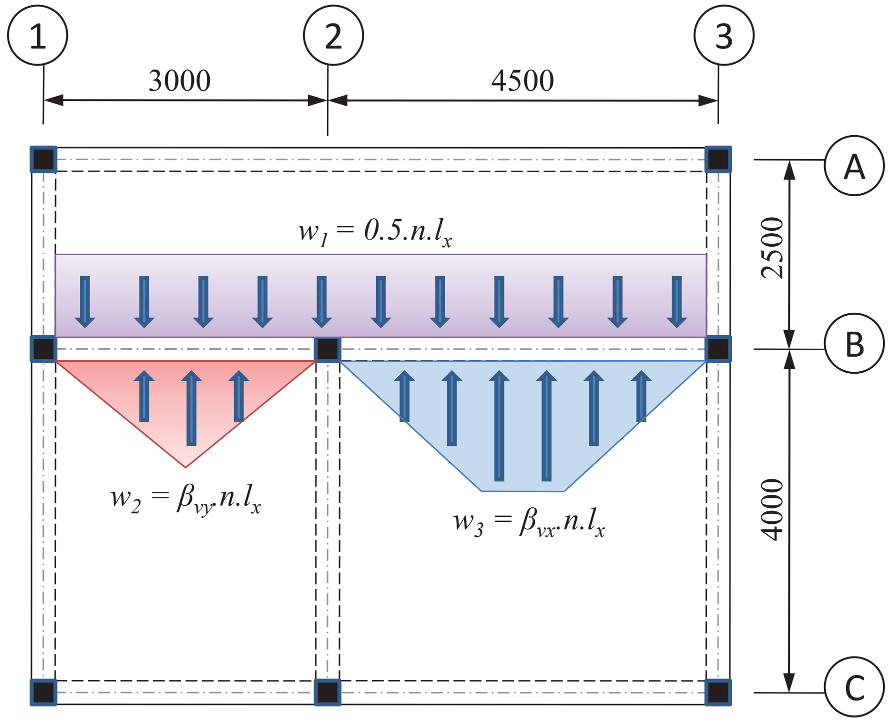
Solution of Example 2.1



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Solution of Example 2.1

Action from slab

w_1 G_k = 0.5 x 5.75 x 2.5 = 7.19 kN/m

w_1 Q_k = 0.5 x 3.00 x 2.5 = 3.75 kN/m

From Table 3.15: BS 8110: Part 1: 1997

Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Two adjacent edges discontinuous									
Continuous edge	0.40		0.47	0.50	0.52	0.54	0.57	0.60	
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26

w_2 G_k = 0.4 x 5.75 x 3.0 = 6.90 kN/m

w_2 Q_k = 0.4 x 3.00 x 3.0 = 3.60 kN/m

w_3 G_k = 0.44 x 5.75 x 4.0 = 10.12 kN/m

w_3 Q_k = 0.44 x 3.00 x 4.0 = 5.28 kN/m

Solution of Example 2.1

Actions on beam

Beam selfweight = 0.20 x (0.5 – 0.15) x 25 = 1.75 kN/m

Span 1-2

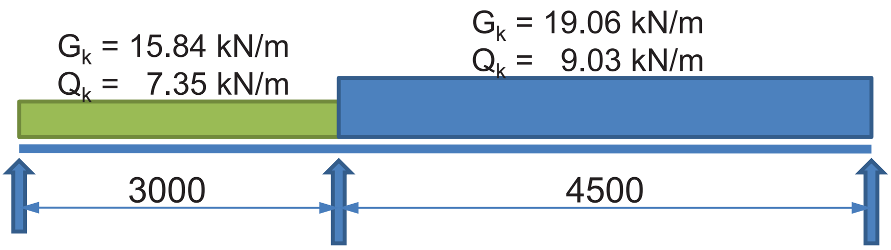
Permanent action,G_k = 7.19 + 6.90 + 1.75 = 15.84 kN/m

Variable action, Q_k = 3.75 + 3.60 = 7.35 kN/m

Span 2-3

Permanent action, G_k = 7.19 + 10.12 + 1.75 = 19.06 kN/m

Variable action, Q_k = 3.75 + 5.28 = 9.03 kN/m



Combination of Actions

- **"Combination of action"** is specifically used for the definition of the magnitude of actions to be used when a limit state is under the influence of different actions.
- For **continuous beam**, **"Load cases"** is concerned with the arrangement of the variable actions to give the most **unfavourable** conditions.
- If there is only one variable actions (e.g. Imposed load) in a combination, the magnitude of the actions can be obtained by multiplying them by the appropriate factors.
- If there is more than one variable actions in combination, it is necessary to identify the leading action(*Qk,1) and other accompanying actions (Qk,i). The accompanying actions is always taken as the combination value.*

Combination of Actions

- Design values of actions, ultimate limit state-persistent and transient design situations

Combination Expression	Permanent actions		Leading variable actions	Accompanying variable actions	
	Unfavourable	Favourable		Main (if any)	Others
Exp. (6.10)	$1.35G_k$	$1.0G_k$	$1.5Q_k$		$1.5\,\psi_{0,i}Q_{k,i}$
Exp. (6.10a)	$1.35G_k$	$1.0G_k$		$1.5\,\psi_{0,1}Q_k$	$1.5\,\psi_{0,i}Q_{k,i}$
Exp. (6.10b)	$0.925\text{x}1.35G_k$	$1.0G_k$	$1.5Q_k$		$1.5\,\psi_{0,i}Q_{k,i}$

Note:
1. Design for either Exp.(6.10) or the less favourable of Exp. (6.10a) and (6.10b)
2. The terms favorable and unfavorable refer to the effect of the action on the design situation under consideration. For example, if a beam, continuous over several spans, is to be designed for largest sagging bending moment it will have to sustain any action that has the effect of increasing the bending moment will be considered unfavorable whilst any action that reduces the bending moment will be considered to be favourable.

Load Cases and Combination

Clause 5.1.3 : MS EN 1992-1-1

- In considering the combinations of actions, the relevant cases shall be considered to enable the critical design conditions to be established at all sections, within the structure or part of the structure considered.
- For **simply supported beam**, the analysis for bending and shear force can be carried out using statically determinate approach. For the ultimate limit state we need only consider the maximum load of $1.35G_k + 1.5Q_k$ on the span.
- While for **continuous beam** a simplification in the number of load arrangements for use in a Country is required, reference is made to its **National Annex**. The following simplified load arrangements are recommended for buildings:

Section 5.1.3 : MS EN 1992-1-1

Load set 1: Alternate or adjacent spans loaded (Continuous Beam)

- Alternate span carrying the design permanent and variable load (1.35*Gk + 1.5Qk), other spans carrying only the design permanent loads (1.35Gk)*

- Any two adjacent spans carrying the design permanent and variable loads (1.35*Gk + 1.5Qk), all other spans carrying only the design permanent load (1.35Gk)*

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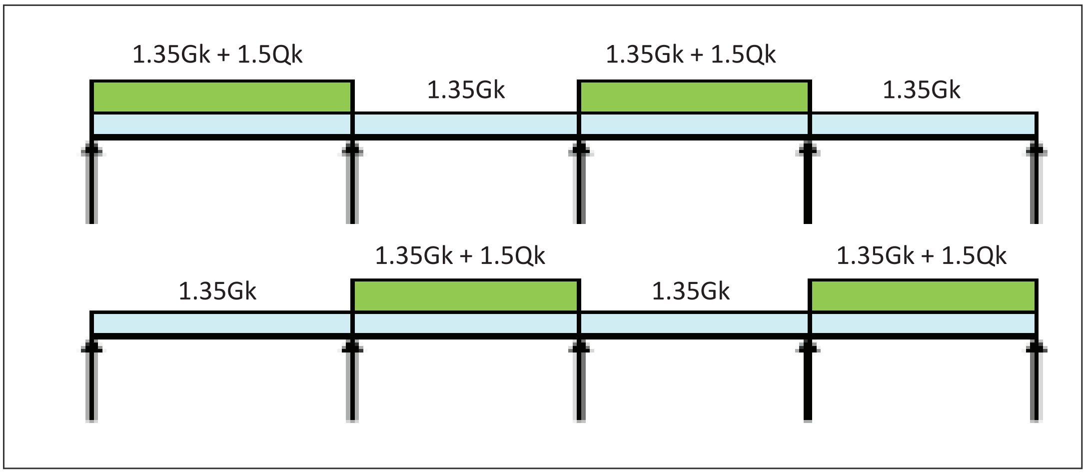
Alternate spans loaded



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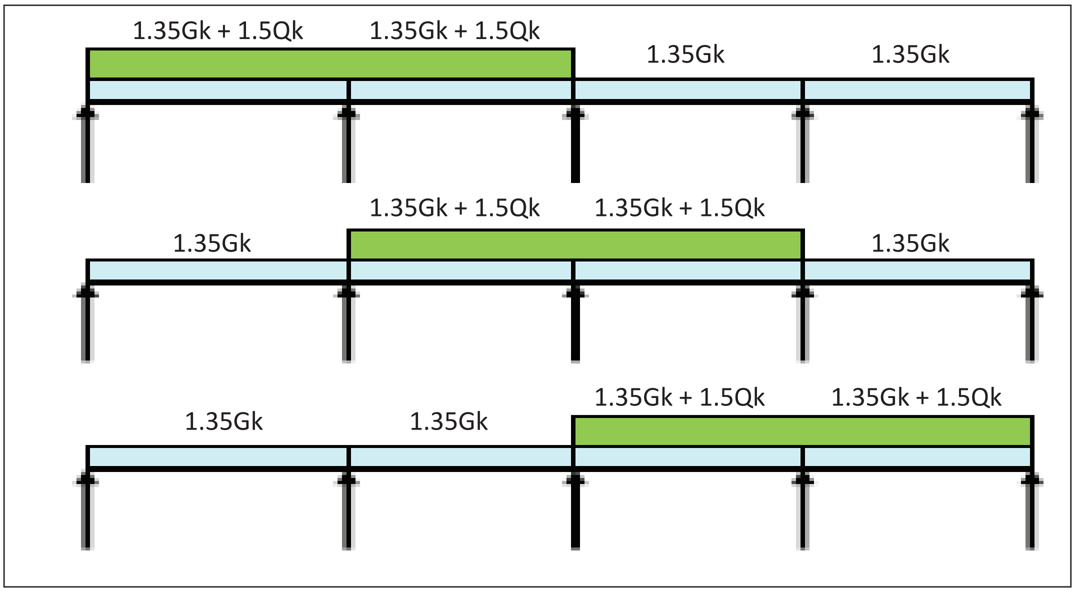
Adjacent Span Loaded



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UK National Annex & Malaysia National Annex

Load set 2: All or alternate spans loaded (Continuous Beam)

- All span carrying the design permanent and variable loads (1.35*Gk+ 1.5Qk)*

- Alternate span carrying the design permanent and variable load (1.35*Gk+ 1.5Qk), other spans carrying only the design permanent loads (1.35Gk)*

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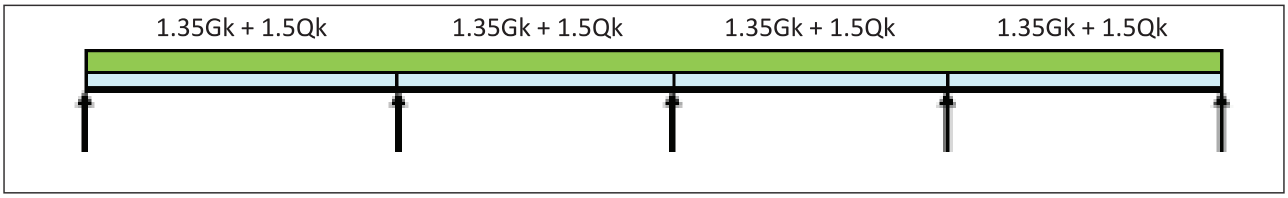
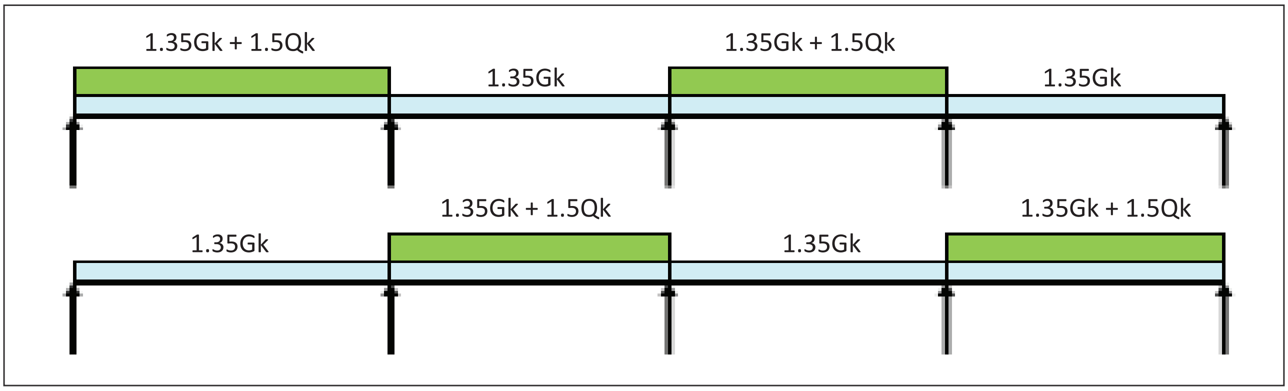
- **All span loaded**



- **Alternate span loaded**



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- The shear force and bending moment diagrams can be drawn for each of the load cases required in the patterns of loading.

- A composite diagram comprising a profile indicating the maximum values including all possible load cases can be drawn; this is known as an **envelope**.

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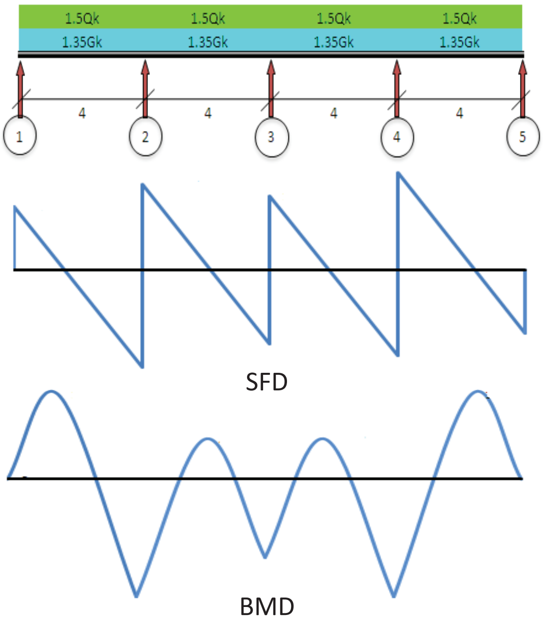
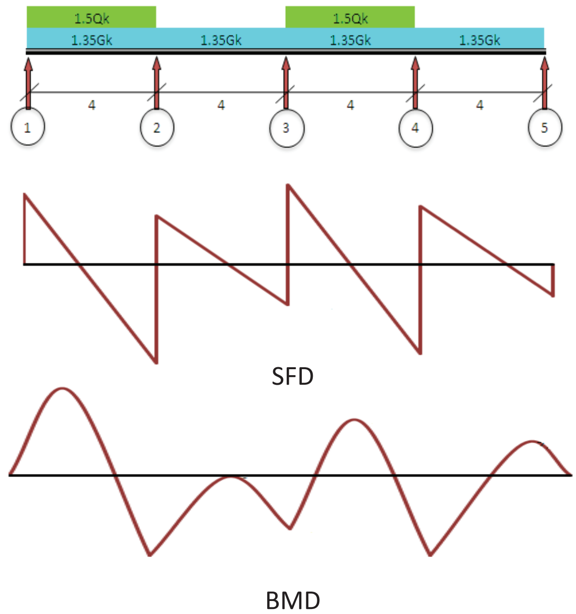
Load Cases and Combination

Load Case 1



Load Case 2



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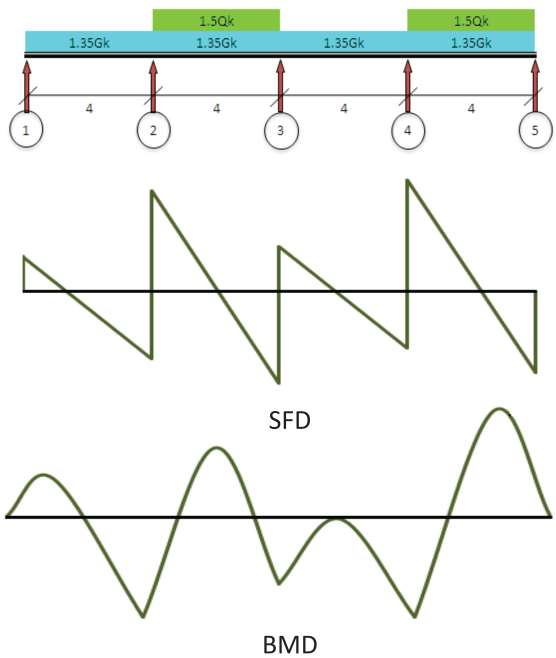
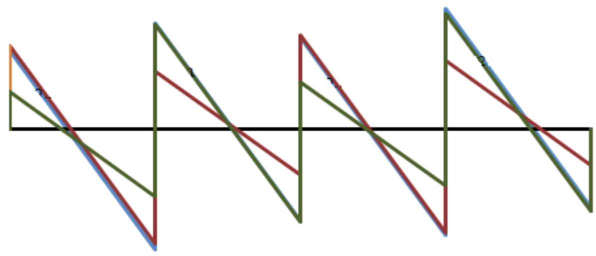
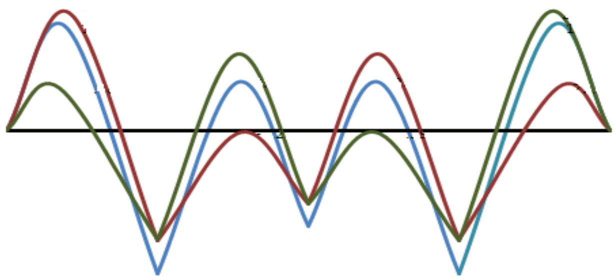
Load Case 3



SHEAR FORCE DIAGRAM ENVELOPE



BENDING MOMENT DIAGRAM ENVELOPE



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Method of Structural Analysis

- Three analysis methods may be used in order to obtain shear force and bending moment for design purposes. There are;
  1. Elastic analysis using moment distribution method (Modified Stiffness Method)
  2. Simplified method using shear and moment coefficient from Table 3.6: BS 8110: Part 1.
  3. Using commercial analysis software such as Staad Pro, Esteem, Ansys, Lusas, etc.

Moment Distribution Method

- Moment distribution method is only involving distribution moments to joint repetitively.
- The accuracy of moment distribution method is dependent to the number repeat which does and usually more than 5 repeat real enough. Right value will be acquired when no more moments that need distributed.
- In general the value is dependent to several factor as :
  - Fixed end moment
  - Carry over factor
  - Member Stiffness Factor (distribution factor)

Fixed end moment (FEM)

- The moment at the fixed joints of a loaded member are called fixed-end moment.
- This moment can be determined from table below, depending upon the type of loading on the member.

Carry over factor (CO)

- The carry-over factor to a fixed end is always 0.5, otherwise it is 0.0.

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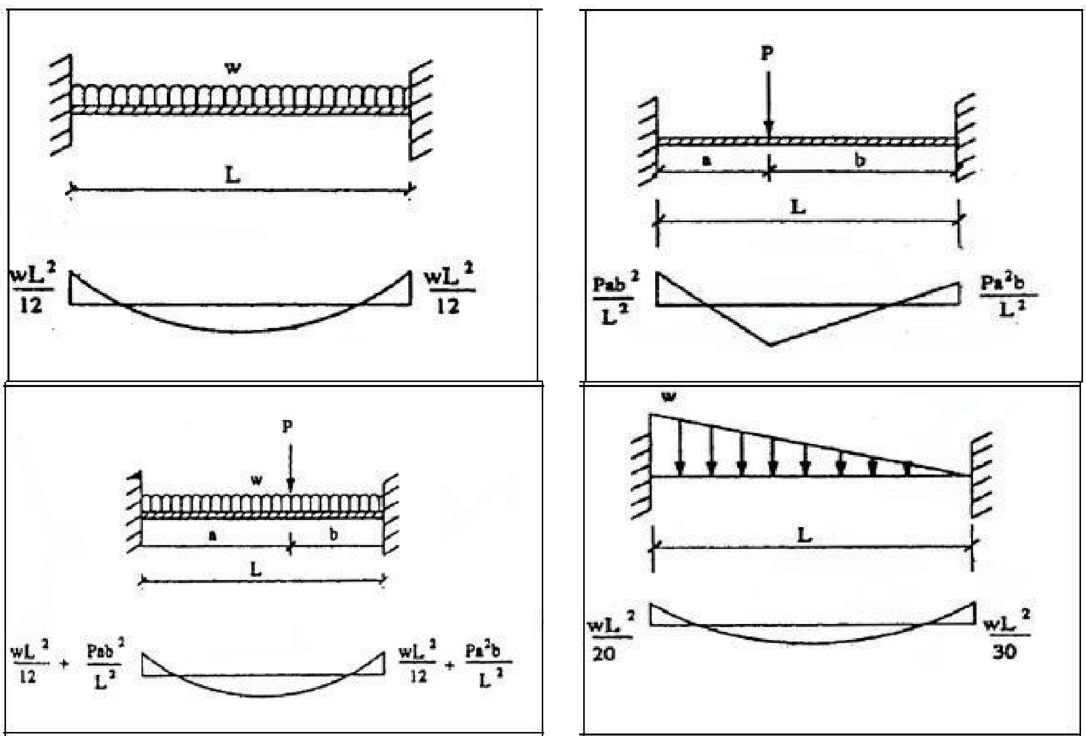
Fixed End Moment



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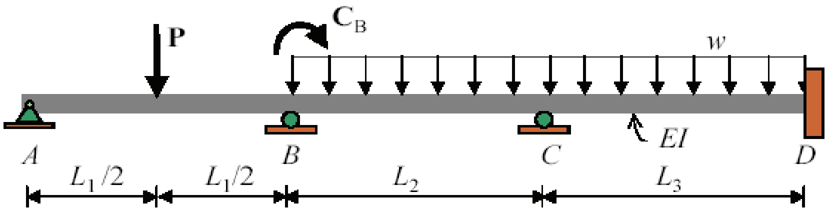
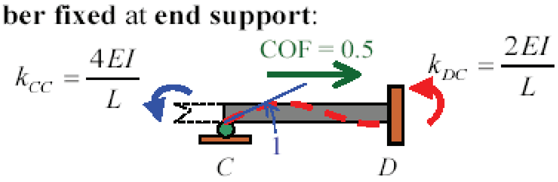
Member Stiffness Factor (*K*) & Carry-Over Factor (COF)

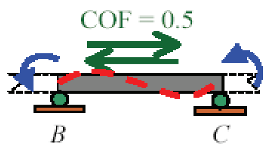


Internal members and far-end member fixed at end support:

$$K = \frac{4EI}{L}$$



$K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$



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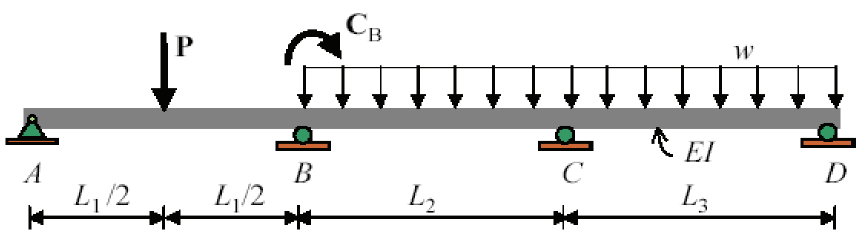
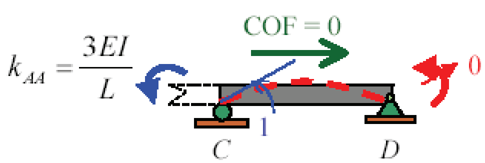
Member Stiffness Factor (*K*)



Far-end member pinned or roller end support:

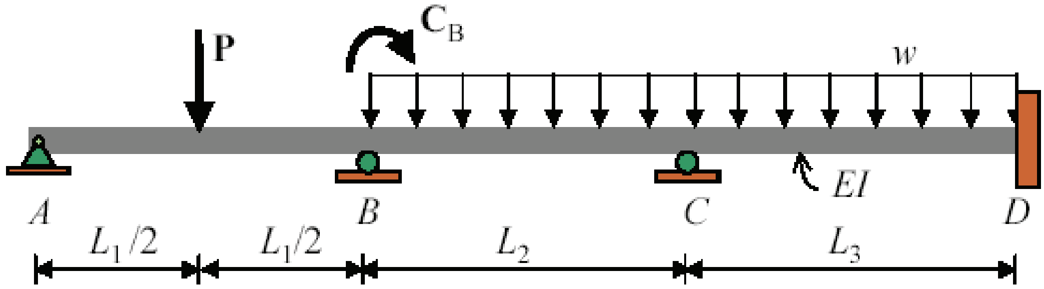
$$K = \frac{3EI}{L}$$



$K_{(AB)} = 3EI/L_1$, $K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$

Moment Distribution Method

Joint Stiffness Factor (*K*)



$K_{(AB)} = 3EI/L_1$ $K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$

$$\boxed{K_{joint} = K_T = \Sigma K_{member}}$$

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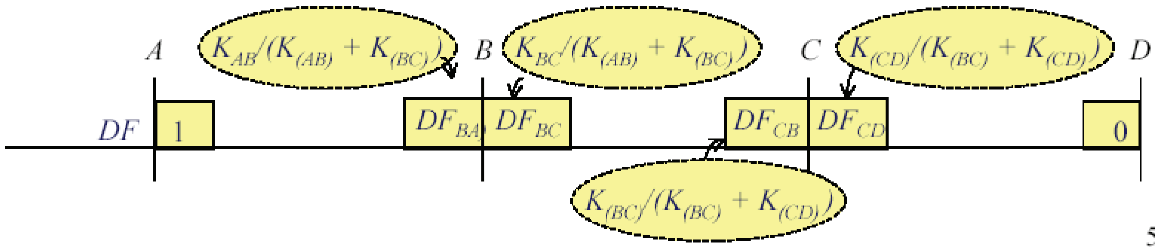
Moment Distribution Method

Distribution Factor (DF)

$$\boxed{DF = \frac{K}{\Sigma K}}$$

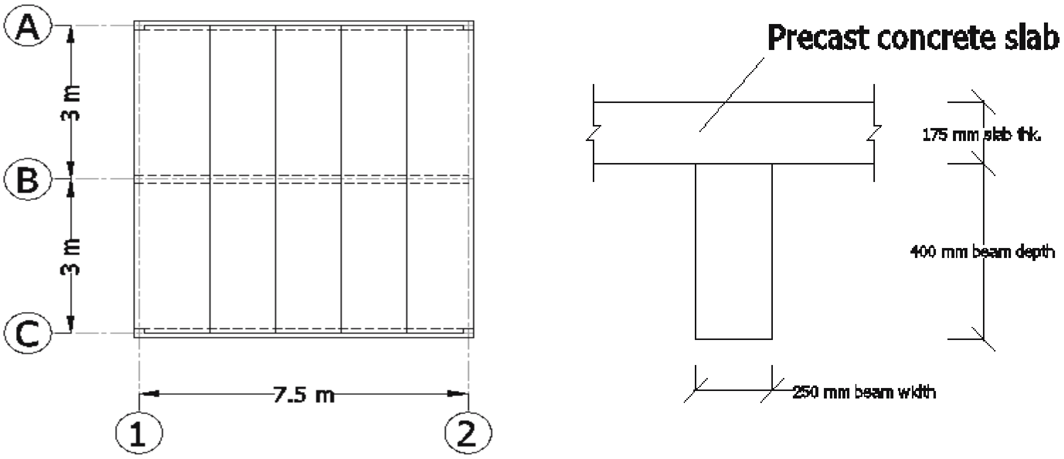
Notes:

- far-end pined (DF = 1)
- far-end fixed (DF = 0)



Example 2.2

A series of 250 x 400 mm reinforced concrete beams spaced at 3 m centres and spanning 7.5 m support a 175 mm thick reinforced concrete slab as shown in Figure 2.1. If the variable floor action is 3 kN/m^2 and the load induced by the weight of concrete is 25 kN/m^3, calculate the maximum shear force and bending moment of beam B/1-2.



Solution of Example 2.2

Slab

Permanent action, g_k from 175 mm slab : 25 x 0.175 = 4.38 kN/m^2

Variable action, q_k : 3.0 kN/m^2

Total ultimate load : 1.35 (4.38) + 1.5(3.0) = 10.41 kN/m^2

Beam

Ultimate load from slab : 10.41 x (1.5x2) = 31.23 kN/m

Beam self-weight, G_k : 25 x 0.4 x 0.25 = 2.5 kN/m

Ultimate load for beam : 1.35 x 2.5 = 3.38 kN/m

Total ultimate load, w : 31.23 + 3.38 = 34.61 kN/m

Maximum shear force = wL/2 = 34.61 x 7.5/2 = **129.79 kN**

Maximum bending moment = wL^2/8 = 34.61 x 7.5^2/8 = **243.35 kNm**

Example 2.3

Figure below shows the first floor layout plan of commercial building. If all beams size are 300 x 500 mm, determine the following;

1. Characteristic permanent and variable action act on the beam 2/A-E if all slab thickness are 150 mm and the brickwall heights is 3m.
2. Shear force and bending moment envelope of beam 2/A-E.

Given the following data;

Variable load on slab	=	4.0 kN/m^2
Finishes, ceiling & services	=	1.5 kN/m^2
Unit weight of concrete	=	25 kN/m^3

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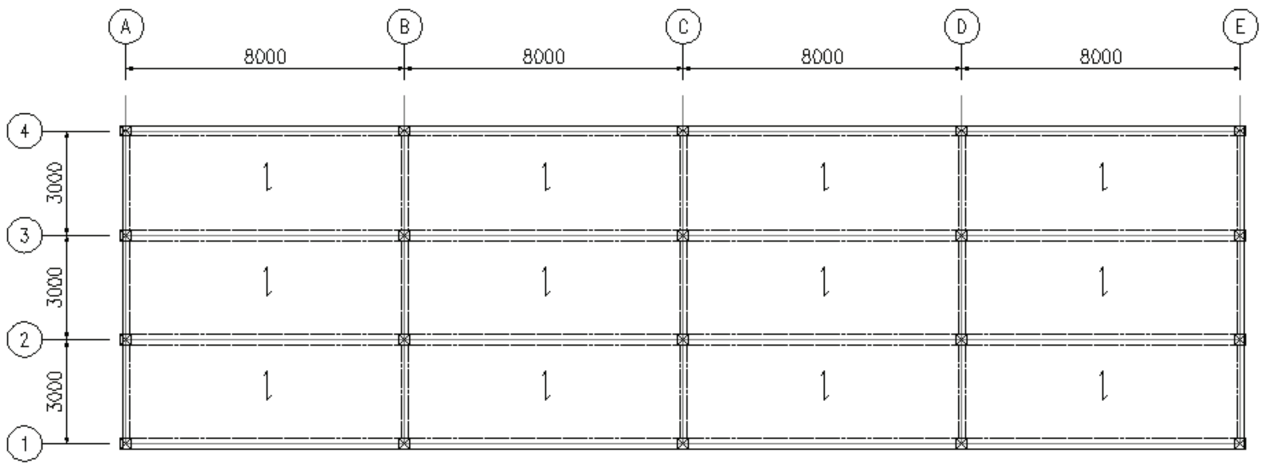
Example 2.3



Solution of Example 2.3

1) Characteristic permanent and variable action act on the beam 2/A-E

Action on slab

Permanent action, Gk

Selfweight of slab	=	0.15 x 25 = 3.75 kN/m^2
Finishes, ceiling and service	=	1.5 kN/m^2
Total permanent action on slab	=	**5.25 kN/m^2**
Variable action, Qk	**=**	**4.0 kN/m^2**

Action on beam

Permanent action, Gk

Load from slab	=	0.5 x 5.25 x 3 = 7.88 kN/m
Beam selfweight	=	(0.3 x (0.5 – 0.15) x 25) = 2.63 kN/m
Brickwall	=	2.6 x 3 = 7.8 kN/m
Total permanent action on beam	=	**18.31 kN/m**

Variable action, Qk

Load from slab	=	0.5 x 4.0 x 3.0 = **6.00 kN/m**

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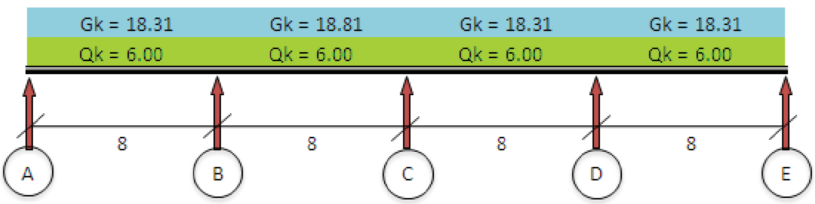
Solution of Example 2.3



2) Shear force and bending moment envelope of beam 2/A-E.

Loading

Span	1.35Gk+1.5Qk (kN/m)	1.35Gk (kN/m)	Load Case		
			Case 1	Case 2	Case 3
A - B	33.72	24.72	33.72	33.72	24.72
B - C	33.72	24.72	33.72	24.72	33.72
C - D	33.72	24.72	33.72	33.72	24.72
D - E	33.72	24.72	33.72	24.72	33.72

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Solution of Example 2.3

Moment of Inertia, I

$I = bh^3/12 = 300 \times 500^3/12 = 3.125 \times 10^9\ mm^4$

Stiffness, K

A-B $K_{AB} = K_{BA} = 3I/L = 3 \times 3.125 \times 10^9/8000 = 1.17 \times 10^6\ mm^3$

B-C $K_{BC} = K_{CB} = 4I/L = 4 \times 3.125 \times 10^9/8000 = 1.56 \times 10^6\ mm^3$

C-D $K_{CD} = K_{DC} = 4I/L = 4 \times 3.125 \times 10^9/8000 = 1.56 \times 10^6\ mm^3$

D-E $K_{DE} = K_{ED} = 3I/L = 3 \times 3.125 \times 10^9/8000 = 1.17 \times 10^6\ mm^3$

Distribution Factor, DF

Joint A & E ~ DF_{AB} & DF_{ED} $= K_{AB} / (K_{AB} + 0) = 1.17 / (1.17 + 0) = 1.0$

Joint B ~ DF_{BA} $= K_{BA} / (K_{BA} + K_{BC}) = 1.17 / (1.17 + 1.56) = 0.43$

DF_{BC} $= K_{BC} / (K_{BA} + K_{BC}) = 1.56 / (1.17 + 1.56) = 0.57$

Joint C ~ DF_{CB} $= K_{CB} / (K_{CB} + K_{CD}) = 1.56 / (1.56 + 1.56) = 0.50$

DF_{CD} $= K_{CD} / (K_{CB} + K_{CD}) = 1.56 / (1.56 + 1.56) = 0.50$

Joint D ~ DF_{DC} $= K_{DC} / (K_{DC} + K_{DE}) = 1.56 / (1.17 + 1.56) = 0.57$

DF_{DE} $= K_{DE} / (K_{DC} + K_{DE}) = 1.17 / (1.17 + 1.56) = 0.43$

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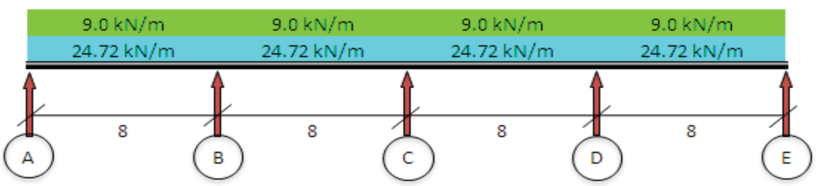
Solution of Example 2.3

Load Case 1



Part	A-B	B-A	B-C	C-B	C-D	D-C	D-E	E-D
DF	1.00	0.43	0.57	0.50	0.50	0.57	0.43	1.00
FEM	-179.83	179.83	-179.83	179.83	-179.83	179.83	-179.83	179.83
Dist	179.83	0.00	0.00	0.00	0.00	0.00	0.00	-179.83
CO	0.00	89.92	0.00	0.00	0.00	0.00	-89.92	0.00
Dist	0.00	-38.54	-51.38	0.00	0.00	51.38	38.54	0.00
CO	0.00	0.00	0.00	-25.69	25.69	0.00	0.00	0.00
Dist	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Dist	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Dist	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Dist	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
End Moment	0.00	231.21	-231.21	154.14	-154.14	231.21	-231.21	0.00

Fix End Moment, FEM

$-M_{AB} = M_{BA} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84\ kNm$

$-M_{BC} = M_{CB} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84\ kNm$

$-M_{CD} = M_{DC} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84\ kNm$

$-M_{DE} = M_{ED} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84\ kNm$

With Wisdom We Explore

Solution of Example 2.3

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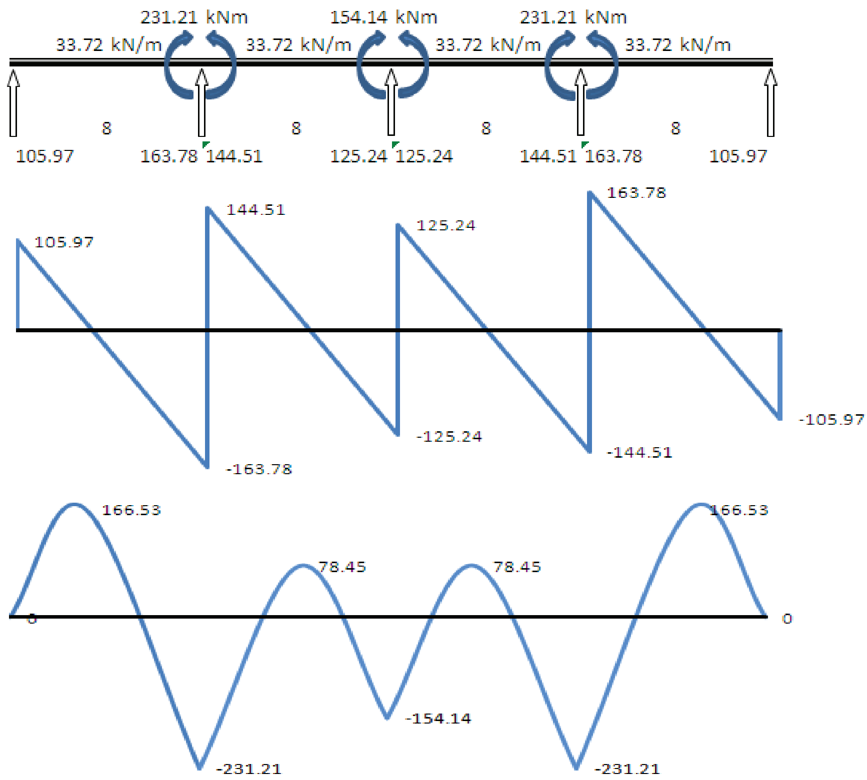

With Wisdom We Explore

Solution of Example 2.3

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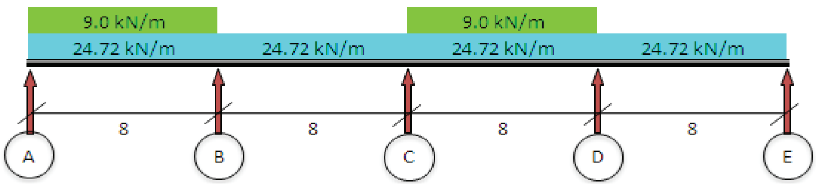
Load Case 2



Part	A-B	B-A	B-C	C-B	C-D	D-C	D-E	E-D
DF	1.00	0.43	0.57	0.50	0.50	0.57	0.43	1.00
FEM	-179.83	179.83	-131.83	131.83	-179.83	179.83	-131.83	131.83
Dist	179.83	-20.57	-27.43	24.00	24.00	-27.43	-20.57	-131.83
CO	0.00	89.92	12.00	-13.71	-13.71	12.00	-65.92	0.00
Dist	0.00	-43.68	-58.24	13.71	13.71	30.81	23.11	0.00
CO	0.00	0.00	6.86	-29.12	15.40	6.86	0.00	0.00
Dist	0.00	-2.94	-3.92	6.86	6.86	-3.92	-2.94	0.00
CO	0.00	0.00	3.43	-1.96	-1.96	3.43	0.00	0.00
Dist	0.00	-1.47	-1.96	1.96	1.96	-1.96	-1.47	0.00
CO	0.00	0.00	0.98	-0.98	-0.98	0.98	0.00	0.00
Dist	0.00	-0.42	-0.56	0.98	0.98	-0.56	-0.42	0.00
CO	0.00	0.00	0.49	-0.28	-0.28	0.49	0.00	0.00
Dist	0.00	-0.21	-0.28	0.28	0.28	-0.28	-0.21	0.00
End Moment	0.00	200.46	-200.46	133.57	-133.57	200.25	-200.25	0.00

Fix End Moment, FEM

$-M_{AB} = M_{BA} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84$ kNm

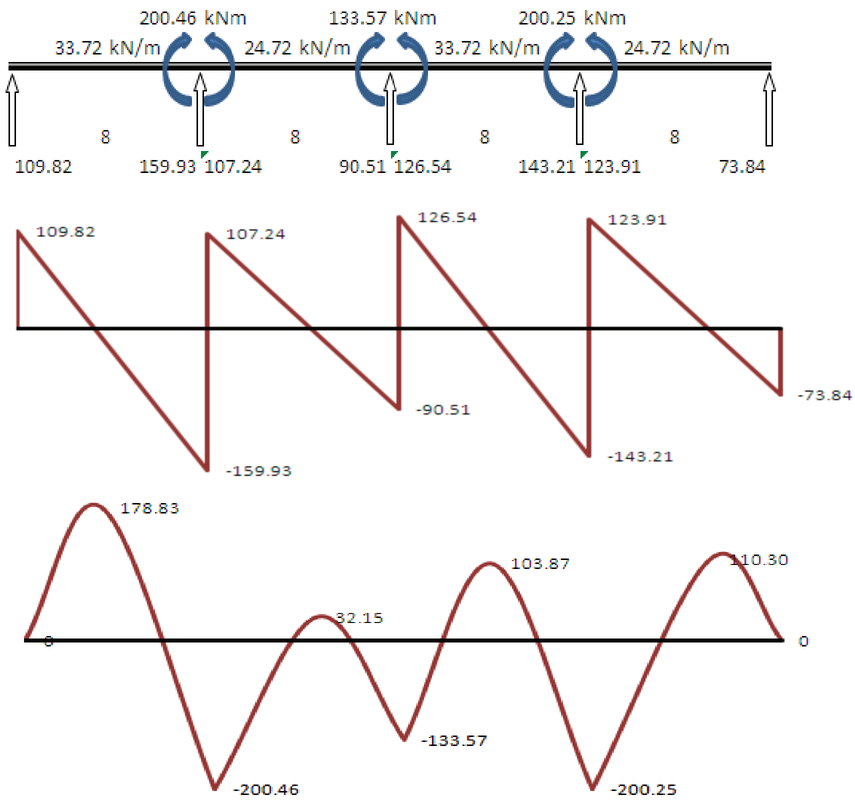
$-M_{BC} = M_{CB} = wL^2/12$
$= 24.72 \times 8^2/12$
$= 131.84$ kNm

$-M_{CD} = M_{DC} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84$ kNm

$-M_{DE} = M_{ED} = wL^2/12$
$= 24.72 \times 8^2/12$
$= 131.84$ kNm

With Wisdom We Explore

Solution of Example 2.3



Solution of Example 2.3

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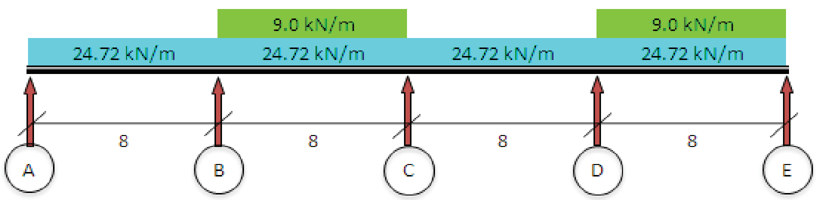
Load Case 3



Part	A-B	B-A	B-C	C-B	C-D	D-C	D-E	E-D
DF	1.00	0.43	0.57	0.50	0.50	0.57	0.43	1.00
FEM	-131.83	131.83	-179.83	179.83	-131.83	131.83	-179.83	179.83
Dist	131.83	20.57	27.43	-24.00	-24.00	27.43	20.57	-179.83
CO	0.00	65.92	-12.00	13.71	13.71	-12.00	-89.92	0.00
Dist	0.00	-23.11	-30.81	-13.71	-13.71	58.24	43.68	0.00
CO	0.00	0.00	-6.86	-15.40	29.12	-6.86	0.00	0.00
Dist	0.00	2.94	3.92	-6.86	-6.86	3.92	2.94	0.00
CO	0.00	0.00	-3.43	1.96	1.96	-3.43	0.00	0.00
Dist	0.00	1.47	1.96	-1.96	-1.96	1.96	1.47	0.00
CO	0.00	0.00	-0.98	0.98	0.98	-0.98	0.00	0.00
Dist	0.00	0.42	0.56	-0.98	-0.98	0.56	0.42	0.00
CO	0.00	0.00	-0.49	0.28	0.28	-0.49	0.00	0.00
Dist	0.00	0.21	0.28	-0.28	-0.28	0.28	0.21	0.00
End Moment	0.00	200.25	-200.25	133.57	-133.57	200.46	-200.46	0.00

Fix End Moment, FEM

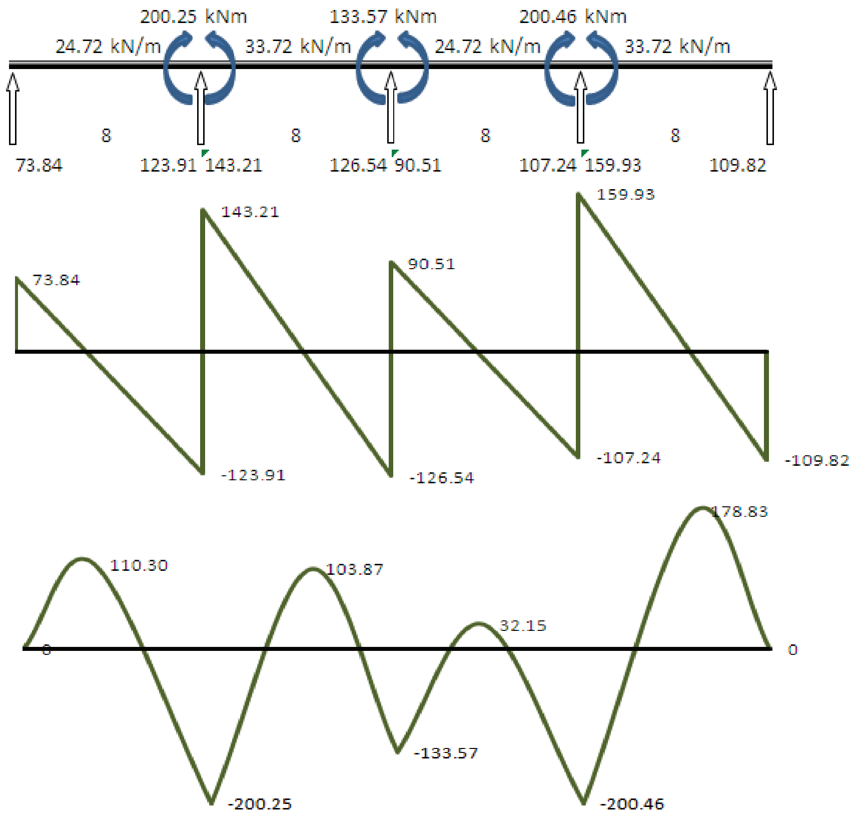
$-M_{AB} = M_{BA} = wL^2/12$
$= 24.72 \times 8^2/12$
$= 131.84$ kNm

$-M_{BC} = M_{CB} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84$ kNm

$-M_{CD} = M_{DC} = wL^2/12$
$= 24.72 \times 8^2/12$
$= 131.84$ kNm

$-M_{DE} = M_{ED} = wL^2/12$
$= 33.72 \times 8^2/12$
$= 179.84$ kNm

Solution of Example 2.3



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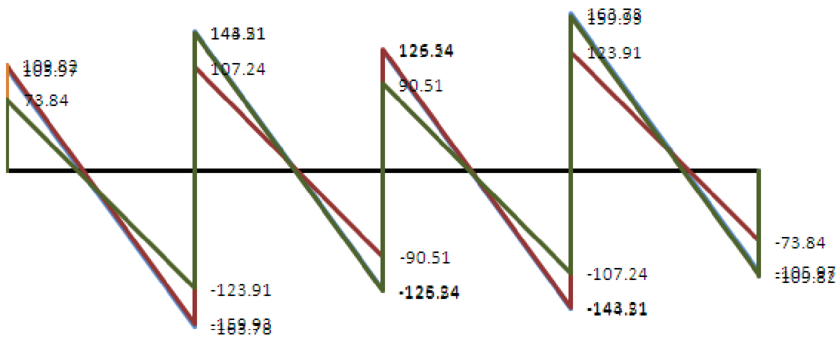
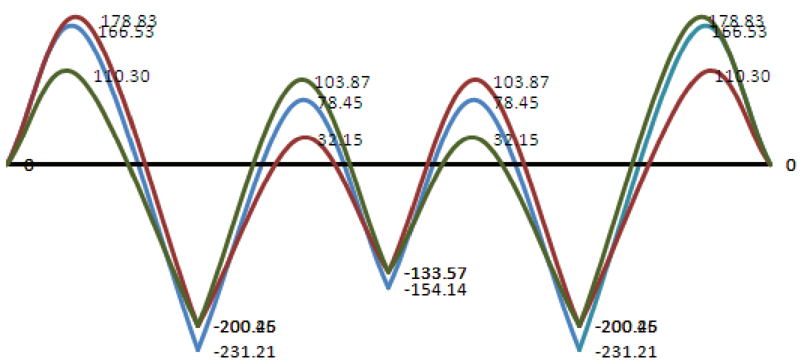
Solution of Example 2.3

SFD Envelope



BMD Envelope



Simplified Method

- The analysis using moment distribution method is time consuming and is more conveniently carried out using standard computer technique.
- Therefore, as a simplification BS 8110 cl. 3.4.3 can be use. Table 3.5 are given in BS 8110 which enable a conservative estimate of shear force and bending moment values to be determined for the design of continuous beam.
- There are conditions which must be satisfied in each case before these tables can be used. They are:

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Simplified Method

- *The beams should be approximately equal span.*
- *The characteristic variable action Qk may not exceed the characteristic permanent action Gk.*
- *Load should be substantially uniformly distributed over three or more spans.*
- *Variation in span length should not exceed 15% of the longest span.*

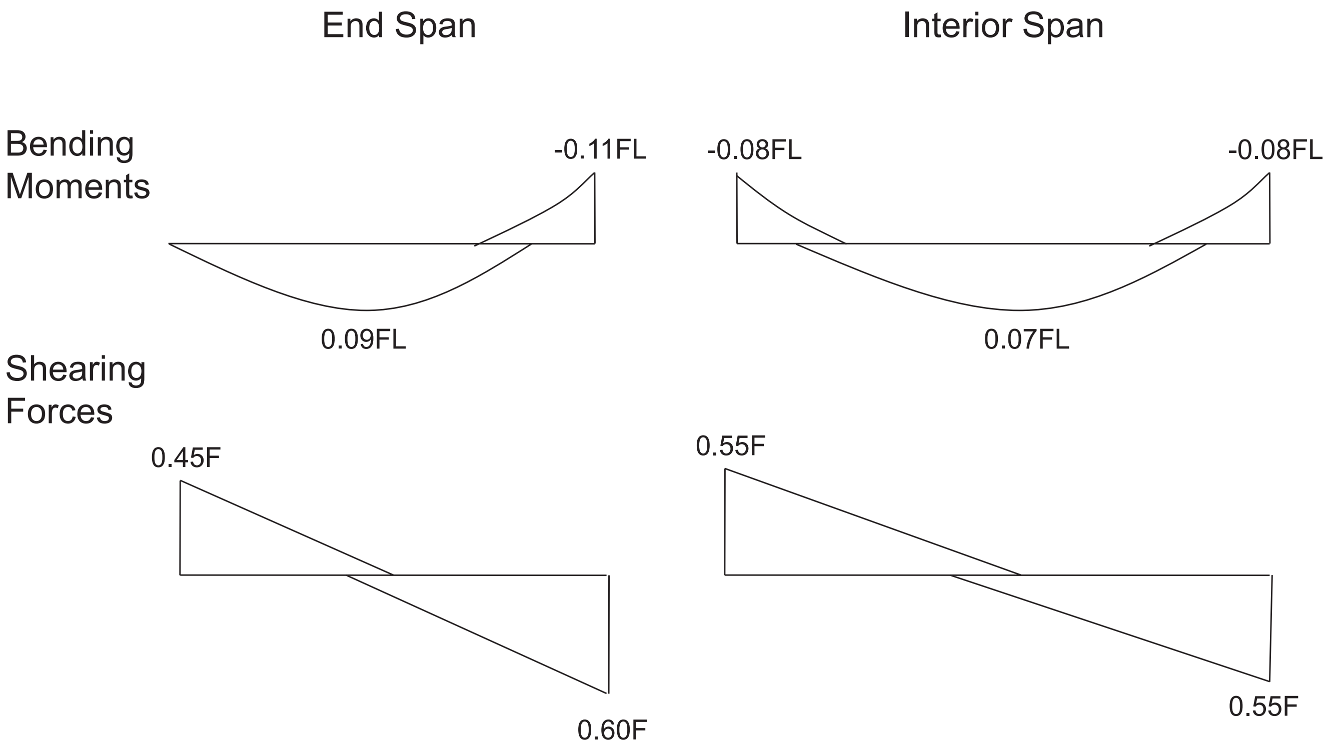
	At outer support	Near middle of end span	At first interior support	At middle of interior spans	At interior supports
Moment	0	$0.09Fl$	$-0.11Fl$	$0.07Fl$	$-0.08Fl$
Shear	$0.45F$	—	$0.6F$	—	$0.55F$

NOTE l is the effective span;
F is the total design ultimate load (1.35Gk + 1.5Qk)

No redistribution of the moments calculated from this table should be made.

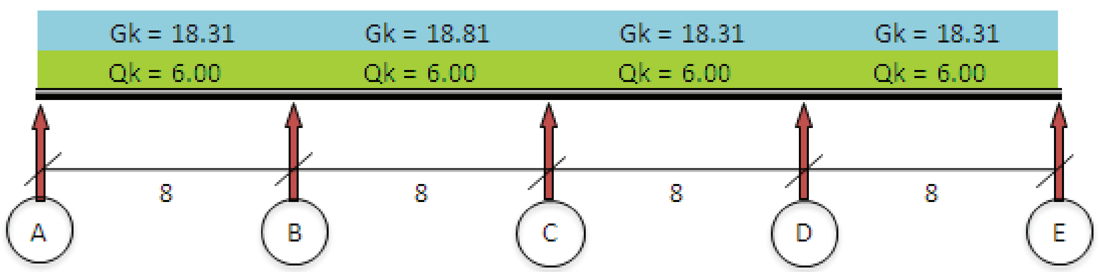
With Wisdom We Explore

Simplified Method



Example 2.4

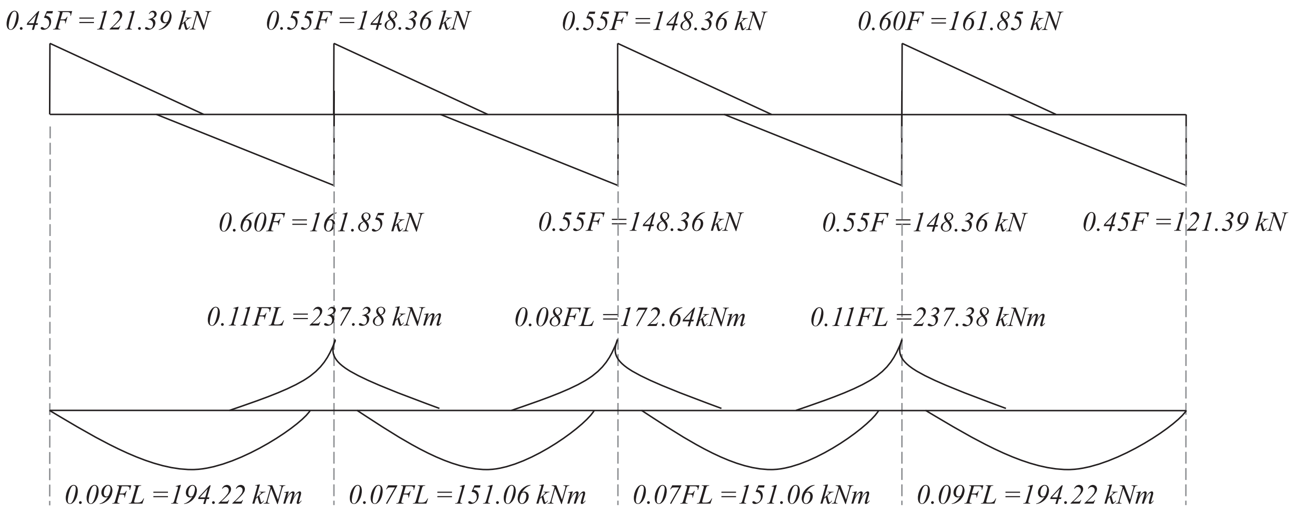
By using simplified method, analyze the beams shown below.



$$F = 1.35G_k + 1.5Q_k$$
$$= 1.35(18.31) + 1.5(6.00) = 33.72\ kN/m \times 8\ m = 269.75\ kN$$

Solution of Example 2.4

Shear force and bending moment diagrams



Moment Redistribution

- Plastic behavior of RC at the ULS affects the distribution of moment in structure.
- To allow for this, the moment derived from an elastic analysis may be redistributed based on the assumption that plastic hinges have formed at the sections with the largest moment.
- From design point of view, some of elastic moment at support can be reduced, but this will increasing others to maintain the static equilibrium of the structure.
- The purpose or moment redistribution is to **reduced the bending moment at congested zone especially at beam-column connection of continuous beam support**. **Therefore, the amount of reinforcement at congested zone also can be reduced then it will result the design and detailing process become much easier**.

Moment Redistribution

- EC2: Section 5.5 permit the moment redistribution with the following requirement;
  - The resulting distribution remains in equilibrium with the load.
  - The continuous beam are predominantly subject to flexural.
  - The ratio of adjacent span should be in the range of 0.5 to 2
- There are other restrictions on the amount of moment redistribution in order to ensure ductility of the beam such as grade of reinforcing steel and area of tensile reinforcement and hence the depth of neutral axis.
  - Class A reinforcement; redistribution should ≤ 20%
  - Class B and C reinforcement; redistribution should ≤ 30%

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Example 2.5

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Based on results obtained from Example 2.3, redistribute 20% of moment at support.

Solution:

Original moment at support B & D = 231.21 kNm

Reduced moment (20%) = 0.8 x 231.21 = 184.97 kNm

Original moment at support C = 154.14 kNm

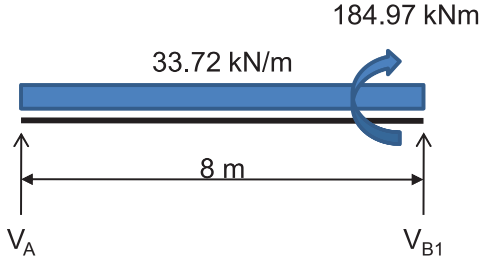
Reduced moment (20%) = 0.8 x 154.14 = 123.31 kNm

With Wisdom We Explore

Solution of Example 2.5

Load Case 1

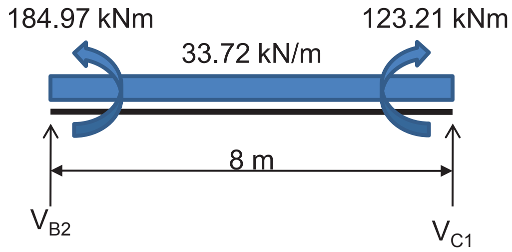
Span A - B



$\Sigma M_B = 0$
$V_A(8) – 33.72(8)^2/2 + 184.97 = 0$
$V_A = 894.07 / 8 = 111.76$ kN

$\Sigma F_y = 0$
$111.76 + V_{B1} – 33.72(8) = 0$
$V_{B1} = 158.0$ kN

Span B - C



$\Sigma M_C = 0$
$V_{B2}(8) – 33.72(8)^2/2 + 123.21 - 184.97 = 0$
$V_{B2} = 1140.8 / 8 = 142.60$ kN

$\Sigma F_y = 0$
$142.60 + V_{C1} – 33.72(8) = 0$
$V_{C1} = 127.16$ kN

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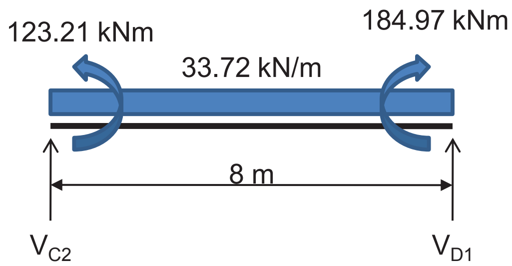
Solution of Example 2.5

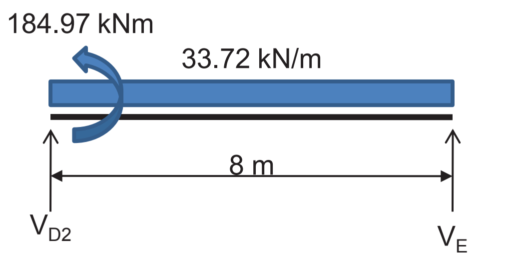
Span C - D



$\Sigma M_D = 0$
$V_{C2}(8) – 33.72(8)^2/2 - 123.21 +184.97 = 0$
$V_{C2} = 1017.28 / 8 = 127.16$ kN

$\Sigma F_y = 0$
$127.16 + V_{D1} – 33.72(8) = 0$
$V_{D1} = 142.60$ kN

Span D - E



$\Sigma M_E = 0$
$V_{D2}(8) – 33.72(8)^2/2 - 184.97 = 0$
$V_{D2} = 1264.01 / 8 = 158.0$ kN

$\Sigma F_y = 0$
$158.0 + V_E – 33.72(8) = 0$
$V_E = 111.76$ kN

With Wisdom We Explore

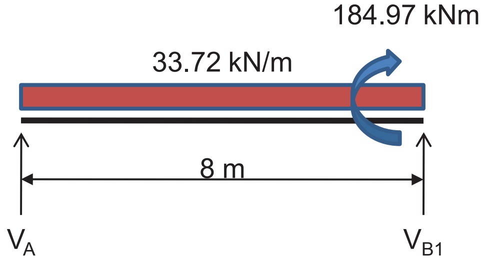
Solution of Example 2.5

Load Case 2

Span A - B

$\Sigma M_B = 0$
$V_A(8) - 33.72(8)^2/2 + 184.97 = 0$
$V_A = 894.07 / 8 = 111.76$ kN

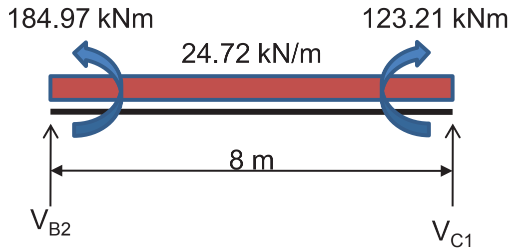
$\Sigma F_y = 0$
$111.76 + V_{B1} - 33.72(8) = 0$
$V_{B1} = 158.0$ kN



Span B - C

$\Sigma M_C = 0$
$V_{B2}(8) - 24.72(8)^2/2 + 123.21 - 184.97 = 0$
$V_{B2} = 852.8 / 8 = 106.60$ kN

$\Sigma F_y = 0$
$106.60 + V_{C1} - 24.72(8) = 0$
$V_{C1} = 91.16$ kN



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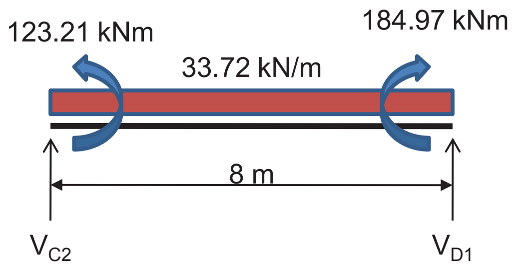
Solution of Example 2.5

Span C - D

$\Sigma M_D = 0$
$V_{C2}(8) - 33.72(8)^2/2 - 123.21 + 184.97 = 0$
$V_{C2} = 1017.28 / 8 = 127.16$ kN

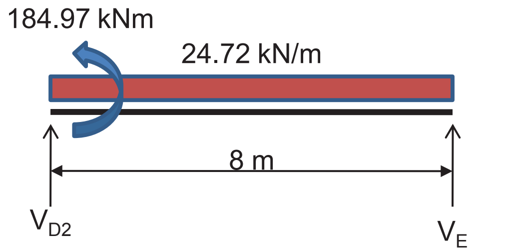
$\Sigma F_y = 0$
$127.16 + V_{D1} - 33.72(8) = 0$
$V_{D1} = 142.60$ kN



Span D - E

$\Sigma M_E = 0$
$V_{D2}(8) - 24.72(8)^2/2 - 184.97 = 0$
$V_{D2} = 976.01 / 8 = 122.0$ kN

$\Sigma F_y = 0$
$122.0 + V_E - 24.72(8) = 0$
$V_E = 75.76$ kN

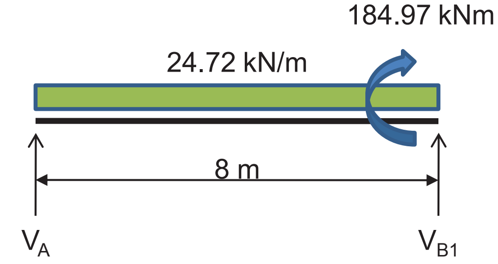


With Wisdom We Explore

Solution of Example 2.5

Load Case 3

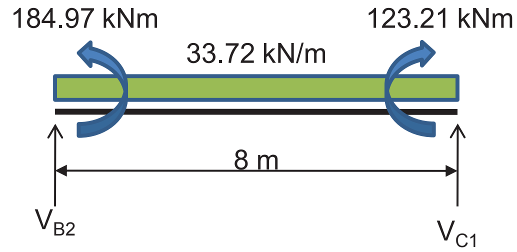
Span A - B



$\Sigma M_B = 0$
$V_A(8) – 24.72(8)^2/2 + 184.97 = 0$
$V_A = 606.07 / 8 = 75.76$ kN

$\Sigma F_y = 0$
$75.76 + V_{B1} – 24.72(8) = 0$
$V_{B1} = 122.0$ kN

Span B - C



$\Sigma M_C = 0$
$V_{B2}(8) – 33.72(8)^2/2 + 123.21 - 184.97 = 0$
$V_{B2} = 1140.8 / 8 = 142.60$ kN

$\Sigma F_y = 0$
$142.60 + V_{C1} – 33.72(8) = 0$
$V_{C1} = 127.16$ kN

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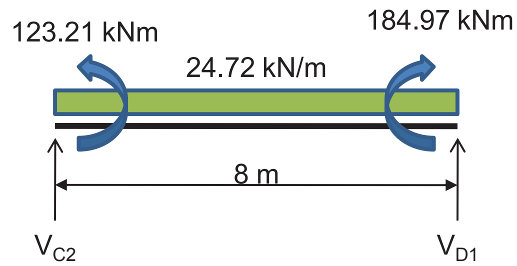
Solution of Example 2.5

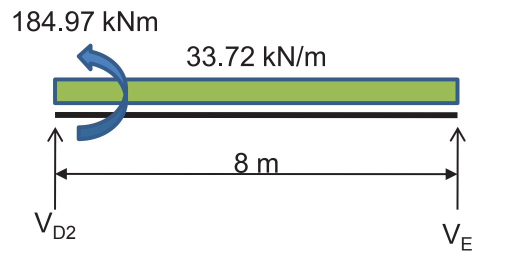
Span C - D



$\Sigma M_D = 0$
$V_{C2}(8) – 24.72(8)^2/2 - 123.21 + 184.97 = 0$
$V_{C2} = 729.28 / 8 = 91.16$ kN

$\Sigma F_y = 0$
$91.16 + V_{D1} – 24.72(8) = 0$
$V_{D1} = 106.60$ kN

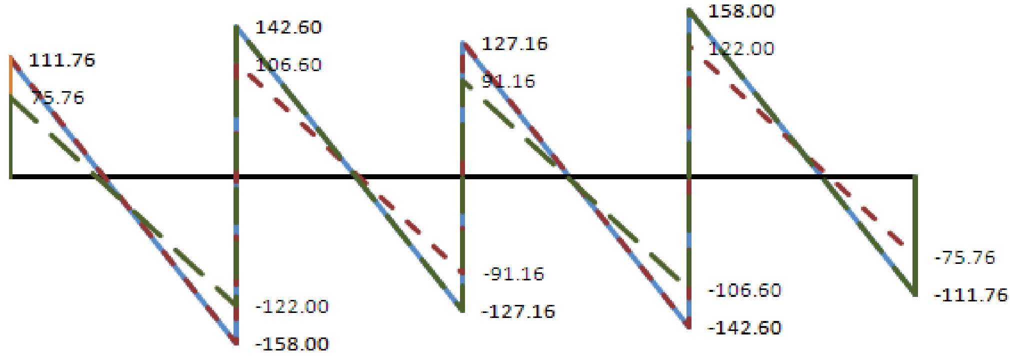
Span D - E



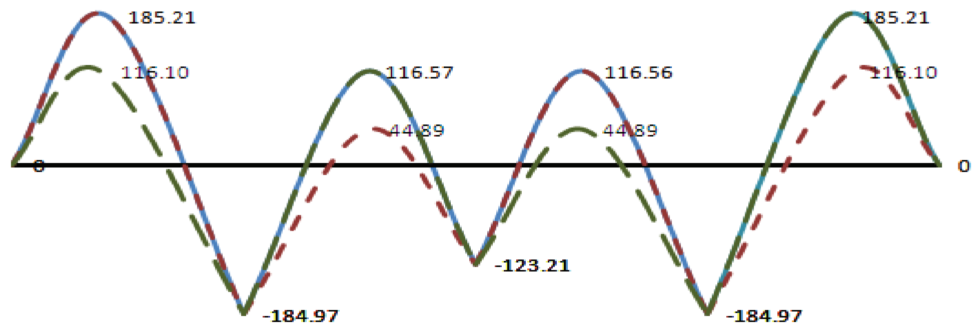
$\Sigma M_E = 0$
$V_{D2}(8) – 33.72(8)^2/2 - 184.97 = 0$
$V_{D2} = 1264.01 / 8 = 158.0$ kN

$\Sigma F_y = 0$
$158.0 + V_E – 33.72(8) = 0$
$V_E = 111.76$ kN

Solution of Example 2.5



SFD Envelope After 20% Redistribution



BMD Envelope After 20% Redistribution